THEME C

Formulator: VERBUNDPLAN; VERBUND – AUSTRIAN HYDROPOWER
Interpretation of measurement results

S. Bonelli, H. F. Felix (will not be presented by the authors)
Delayed response analysis of temperature effect

A. Carrere, C. Noret-Duchene
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Interpretation of measurement results

6th INTERNATIONAL BENCHMARK WORKSHOP
ON NUMERICAL ANALYSIS OF DAMS
October 17-19, 2001, Salzburg - Austria

Ad Hoc Committee on Computational Aspects of Analysis and Design of Dams

Theme C

INTERPRETATION OF MEASUREMENT RESULTS

Formulator: **VERBUNDPLAN**
VERBUND - AUSTRIAN HYDROPOWER

1 INTRODUCTION

1.1 Objective of this Benchmark

Theme C concerns the analysis of the measurement results for the radial crest displacements of Schlegeis Arch Dam.
The data provided are:

- Simplified 3D FE-model of the dam,
- Time histories of water level, ambient temperature and concrete temperatures for the period 1992 to 2000,
- Time history of radial crest displacement for the period 1992 to 1998.

The requested results are:

- The formula and the parameters governing the radial crest displacements in terms of water level (actual and history) and temperature and
- A prognosis of the radial crest displacements for the period 1999 to 2000

With the formula and the parameters it should be possible to separate temperature effects and the influence of the water load and to identify the elastic, viscoelastic and plastic portions of the observed radial crest displacements.
The analysis can be based on the results of a structural analysis of the dam by means of the provided FE-model, on a statistical analysis of the instrument readings or on a combination of structural and statistical analysis.

1.2 General Information about Schlegeis Dam

1.2.1 Description of Dam

Schlegeis arch dam was constructed between 1969 and 1971. It is a double curvature arch/ gravity dam with a crest length/dam height ratio of 5.5.

The foundation of the dam consists of fairly uniform granitic mica gneiss. Its plane of schistosity strikes approximately parallel to the right bank abutment and has a very steep dip towards downstream. The dam consists of 43 blocks, each 17 m wide. It is provided with 4 horizontal inspection galleries and the base gallery located at the foundation rock (see Figure 1 and 2).

The upstream grout curtain originally consisted of a vertical main curtain beneath the dam base extending to a depth of 50m, and a shallower secondary grout curtain extending from the base gallery and inclined towards the upstream (see Figure 2).
The main technical data are:

Crest elevation	1783m a.s.l.
Dam height	131m
Crest length	725m
Crest thickness	9m

Base thickness	34m
Concrete volume	960 000m^3
Top water level	1782m a.s.l.
Drawdown level	1680m a.s.l.
Live storage	127 hm^3

When top storage level was first reached in 1973, seepage rates of 250l/s were measured in the middle part of the base gallery. This water inflow was due to cracking in the foundation rock caused by tensile stresses. To reduce this amount of seepage water a cut off wall was built in total of eleven blocks. Since then the seepage rate at top storage level has not exceeded 25 l/s.

1.2.2 Instrumentation, Measurement

Because of the large span in relation to its height the dam is monitored with a large number of instruments. The most important surveillance instruments are five shafts with pendulums (see Figure 1). In the following only the measurements concerning the Benchmark Workshop are described.

- **Pendulums**

In this benchmark we will consider the pendulums in block 0 only. The radial movement of the crest of the dam relative to the point of reference 80 m below foundation has to be analysed.

Because of the vertical curvature of the dam it was necessary to install two pendulums (one inverted pendulum fixed 80 m below the dam base and one direct pendulum fixed at the dam crest) to obtain the crest displacement. The provided values for this benchmark-workshop are the crest displacements for the period 1992 to 1998, one value per day (at 09:00 MET).

- **Concrete Temperatures**

Concrete temperatures are measured daily in block 0 in two horizons, elevation 1750.65 m and 1677.15 m. In each horizon three thermometers are installed (see Figure 2 for details). The provided values for this benchmark-workshop are those for 09:00 MET, for the period 1992 to 2000.

- **Air Temperature**

Air temperatures are measured near the dam crest at each full hour. This 24 values per day (from 00:00 until 23:00) are used to calculate unweighted arithmetic mean values which are provided for the period 1992 to 2000.

- **Waterlevel**

For this Benchmark Workshop the waterlevel at 09:00 MET is provided every day for the period 1992 to 2000.

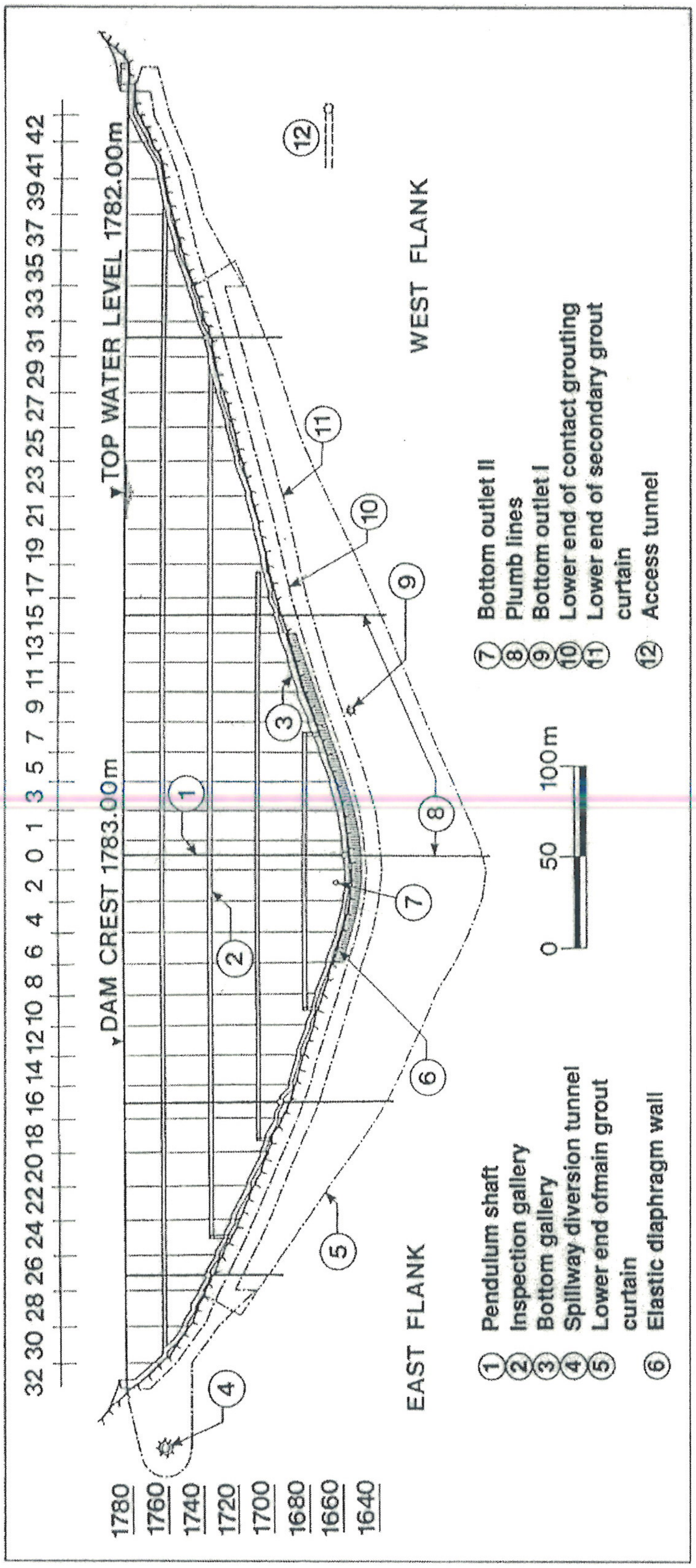

Figure 1: Overview of the Schlegeis Arch Dam

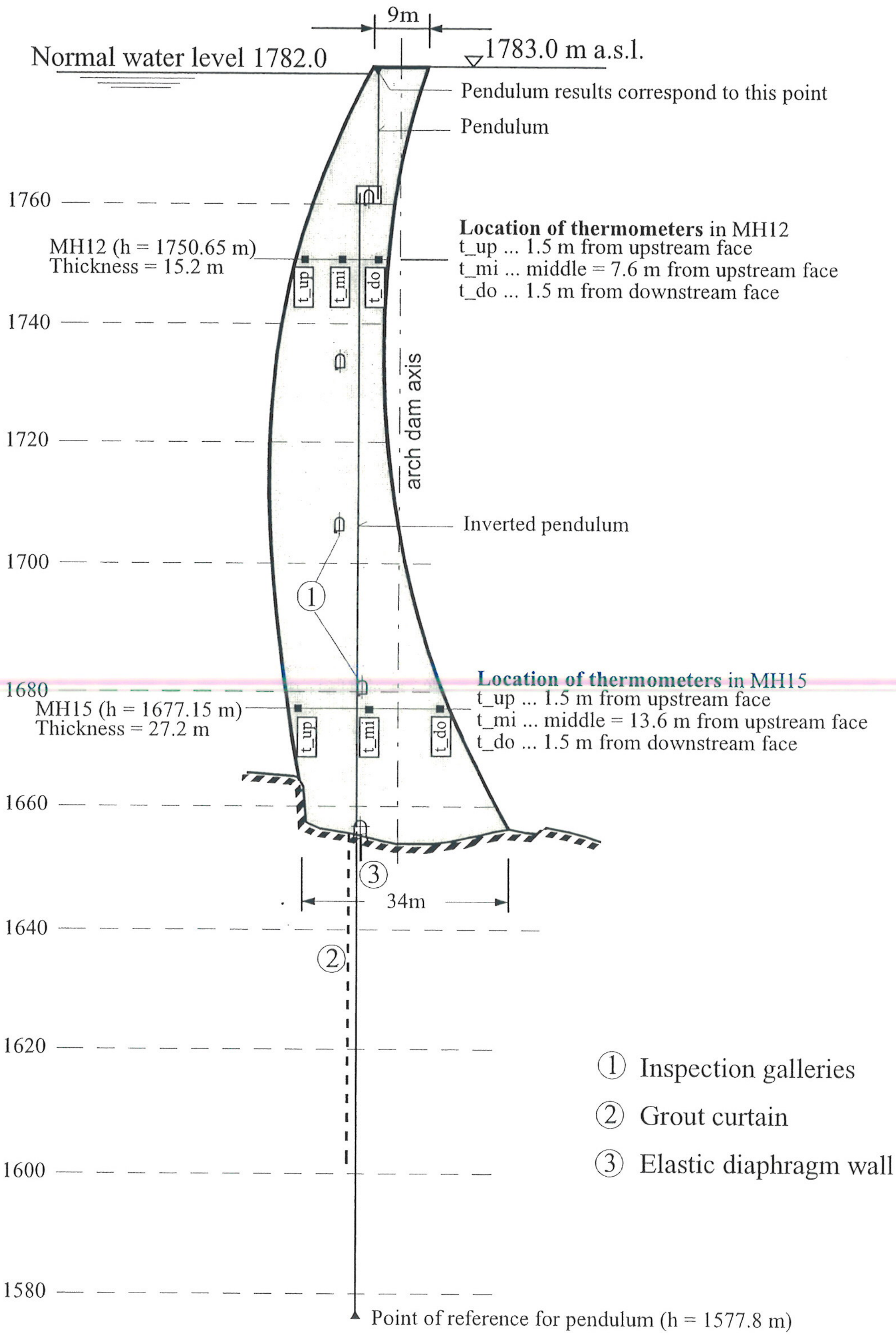

Figure 2: Central Cross Section

2 ANALYSIS DATA

The analysis data are collected in the following files:

MONIT.TXT	measured data	(see Section 2.1)
BRICK.TXT	brick elements (20 node elements)	(see Subsection 2.2.1)
C3D27.TXT	brick elements (27 node elements)	(see Subsection 2.2.1)
NODE.TXT	nodes, x-, y-, z-coordinates	(see Subsection 2.2.1)
N_COINC.TXT	coincident nodes in dam-rock interface	(see Section 2.2)
WEDGE.TXT	wedge elements (15 node elements)	(see Subsection 2.2.1)

2.1 Monitoring Data

The monitoring data are provided in textfile 'MONIT.TXT'. The file represents a table with 11 columns and 3288 data lines plus three heading lines. The table is sorted by date. Data records start at 1992-01-01 and end 2000-12-31. The recordings of pendulum-measurements are removed from the dataset from 1999-01-01 until 2000-12-31 (These values should be prognosticated).

2.1.1 Description of Columns in 'MONIT.TXT'

Please, see also the description under point 1.2.2
The unit for all temperatures is degrees Celsius.

- **Column 1** (COUNT)
  This values represent a counter, starting with '1' in the first data line. It increments by 1 each day.
- **Column 2** (DATE)
  Here you can find the date of monitoring in the format YYMMDD (f. e. 001222 means 2000-December-22).
- **Column 3** (W_LEVEL)
  In this column you find the 9.00 (MET) values for the water level in meter a. s. l.
- **Column 4** (PENDULUM)
  The pendulum values for block 0(radial crest displacement), measured at 9.00 MET in millimeter. This values are provided until 1998,December 31 only.
- **Column 5** (T_H12_UP)
  Concrete temperature in horizon MH12. Location is 1.5 m from upstream face (see figure 2, t_up).
- **Column 6** (T_H12_MI)
  Concrete temperature in horizon MH12. Location is middle of cross-section (see figure 2, t_mi).

- **Column 7** (T_H12_DO)
  Concrete temperature in horizon MH12. Location is 1.5 m from downstream face (see figure 2, t_do).
- **Column 8** (T_H15_UP)
  Concrete temperature in horizon MH15. Location is 1.5 m from upstream face (see figure 2, t_up).
- **Column 9** (T_H15_MI)
  Concrete temperature in horizon MH15. Location is middle of cross-section (see figure 2, t_mi).
- **Column 10** (T_H15_DO)
  Concrete temperature in horizon MH15. Location is 1.5 m from downstream face (see figure 2, t_do).
- **Column 11** (T_AIR_M)
  Mean Value of air temperature (see also 1.2.2)

2.2 FE-Model (optional)

It is the choice of each participant either to perform a FE-analysis for the prediction of the crest displacements or to perform a statistical analysis with the available data set or to do both, static and statistical analysis.

If it is decided to perform a FE-analysis the necessary data for the model under this point of the description can be found.

A linear or nonlinear version of the FE model can be used. The nonlinear version is realized by using the provided double nodes to include "contact-elements" between dam and rock. If the linear version is used, a simple way to do that is to join the double nodes by suitable conditions (the file 'N_COINC.TXT' with the corresponding coincident nodes is included in the data files for this reason).

2.2.1 Description of Provided FE-Model

- **Basic data**

The mesh has about 6.000 nodes resulting into around 18.000 degrees of freedom. The nodes (node numbers, x-, y-, z-coordinates) can be found in file 'NODE.TXT'. Please note, that from z-coordinates 1.000 m is subtracted to get more convenient numbers.

	Elements	Nodes
Dam	246	1369 + 108
Rock	896	4713 + 108
Interface	54	

Table 1: Basic Data - Finite Element Mesh

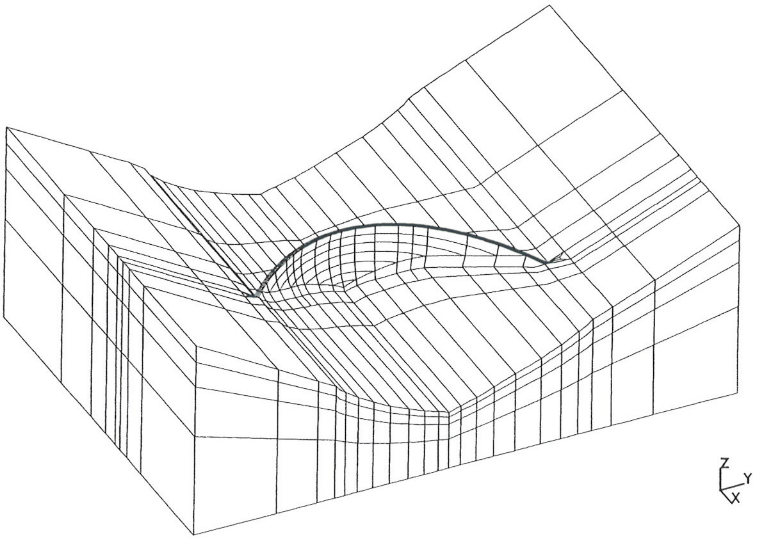

Figure 3: Downstream View of the Finite Element mesh

Three elements are foreseen in radial direction for the dam (see Figure 7).
The elements of the FE-mesh are quadratic, isoparametric 20 and 15 node volume elements (see Figure 4).

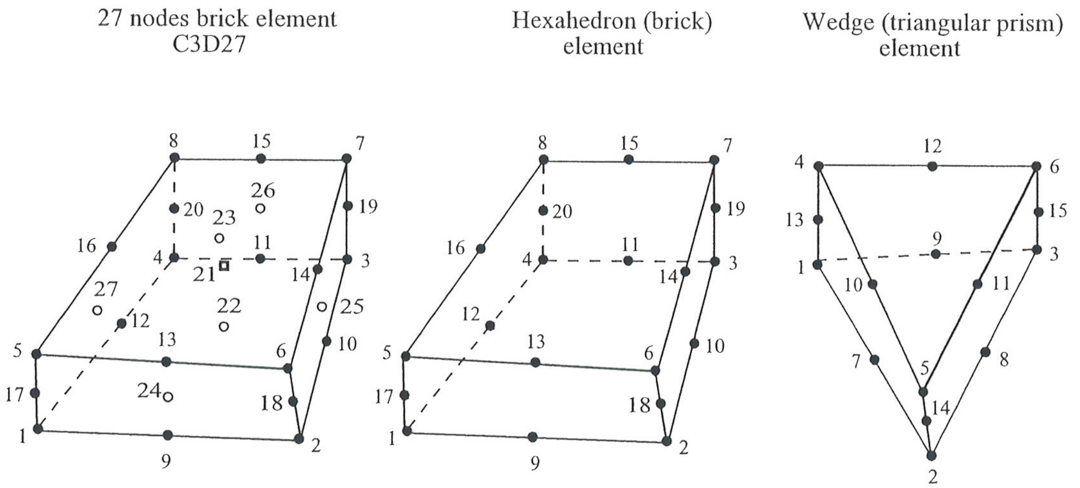

Figure 4: Node Order of the Brick and Wedge Element

There are three files describing elements. In file 'BRICK.TXT' the element number followed by 20 node numbers ordered by the scheme in Figure 4 can be found. File 'WEDGE.TXT' gives element numbers plus 15 node numbers for wedge elements. File 'C3D27.TXT' is a special format (for example used by ABAQUS) and describes brick elements with 27 nodes (20 nodes like normal hexahedron element plus 6 midsurface nodes plus 1 midvolume node). Normal brick elements can be created from this elements by deleting the last seven node numbers for each element.

- **Material Parameters**

For the rock foundation an orthotropic material behaviour is assumed. The rock is assumed massless during analysis steps. The plane of schistosity is shown in Figure 5. The rock consists of Granite-Gneiss with a schistosity of 340/75 Degrees related to Gauss-Krüger-System (29.98/75 Degrees relative to local Y-axis).

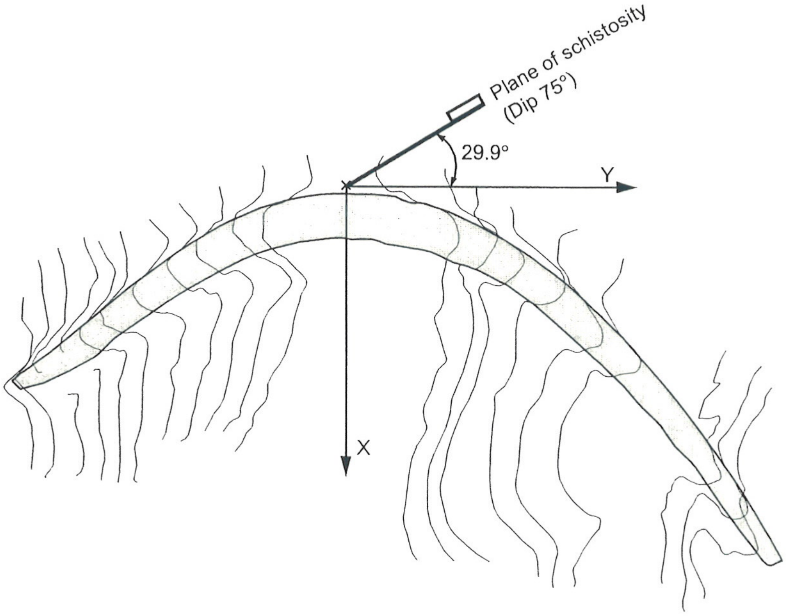

Figure 5: Description of Schistosity

The dam is modeled with isotropic material behaviour.

	Rock	Concrete
Young's modulus E [GPa] (for rock E_{II})	30	25
Young's modulus E [GPa] (for rock $E\perp$)	10	
Poisson ratio ν	0.17	0.17
Density [kg/m³]		2400
α_T		8.10^{-6}

Table 2: Material Parameters

The joint behaviour is described by the normal to shear stress relation (only for nonlinear analysis) shown in Figure 6.

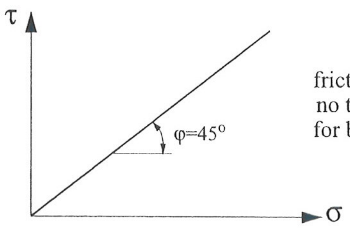

Figure 6: Friction Law

- **Loading**

The loadings to be considered are upon participants choice. Some arbitrary examples are given:

Selfweight of the dam

Water load for full or partially full reservoir

Uplift pressure for full or partially full reservoir

Temperature.

It is suggested to apply the selfweight of the dam on individual concrete blocks (open block joints). Water load and uplift pressure can be as indicated in Figure 7.

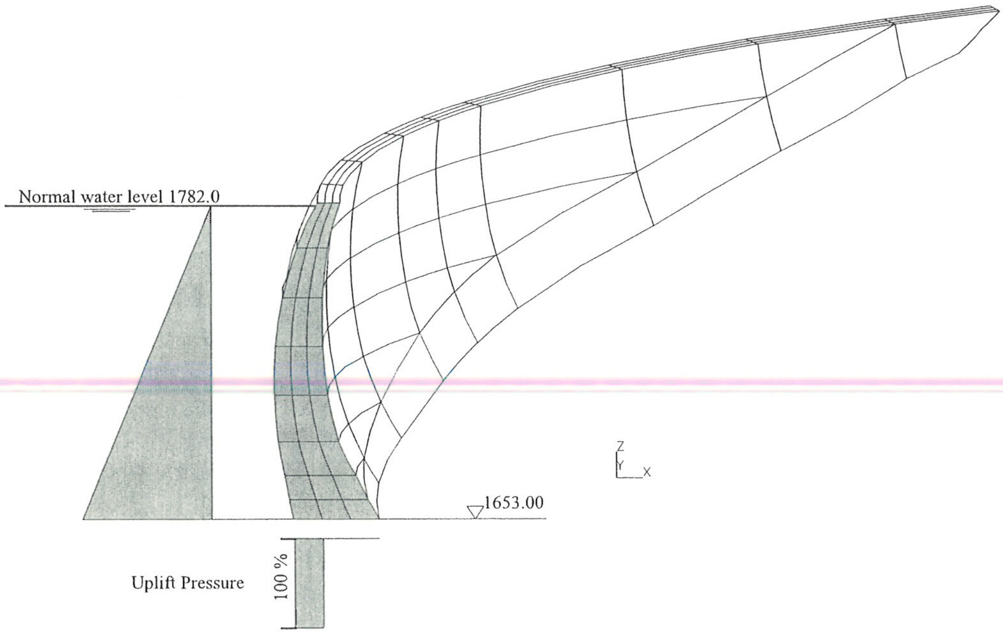

Figure 7: Dam Model, Water and Uplift Loading

In Figure 8 the upstream face of the dam is displayed. For each element the element number and the relevant face number is represented (face numbers are related to definition in ABAQUS).
In Figure 9 the element numbers and face numbers relevant for upstream definition are shown.

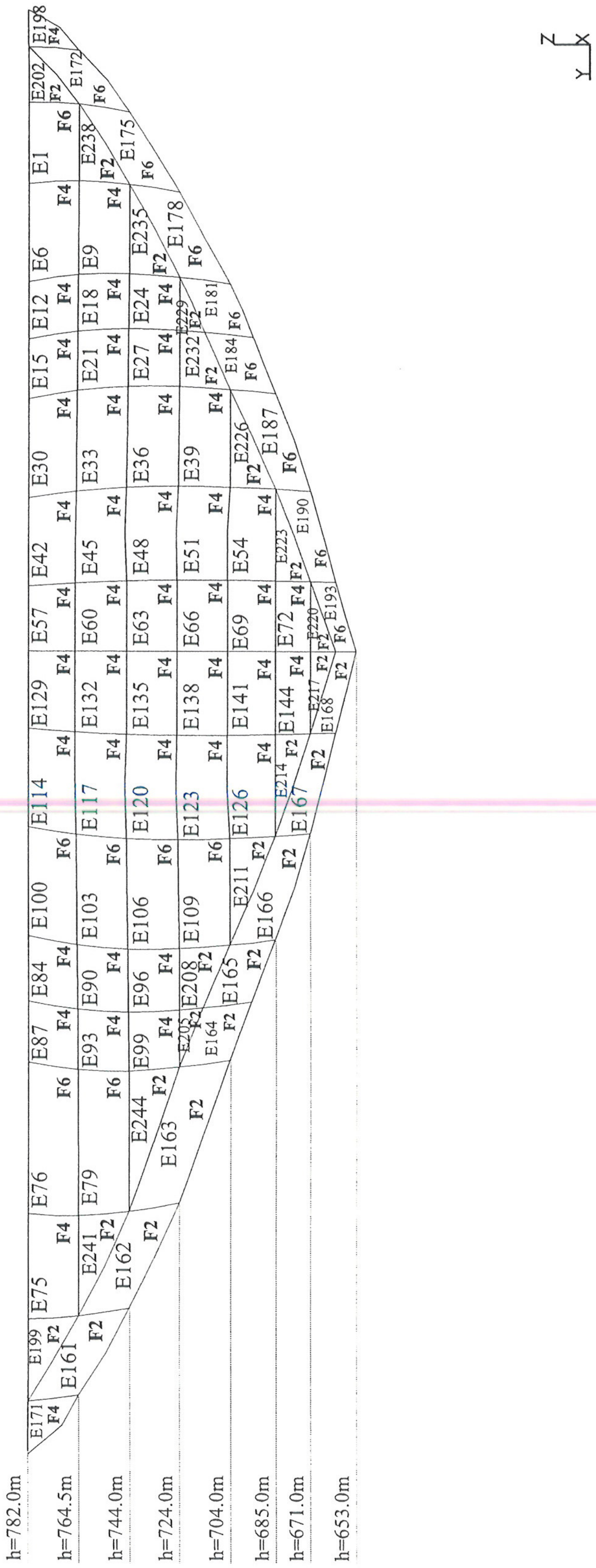

Figure 8: Element Numbers and Faces on Upstream Face

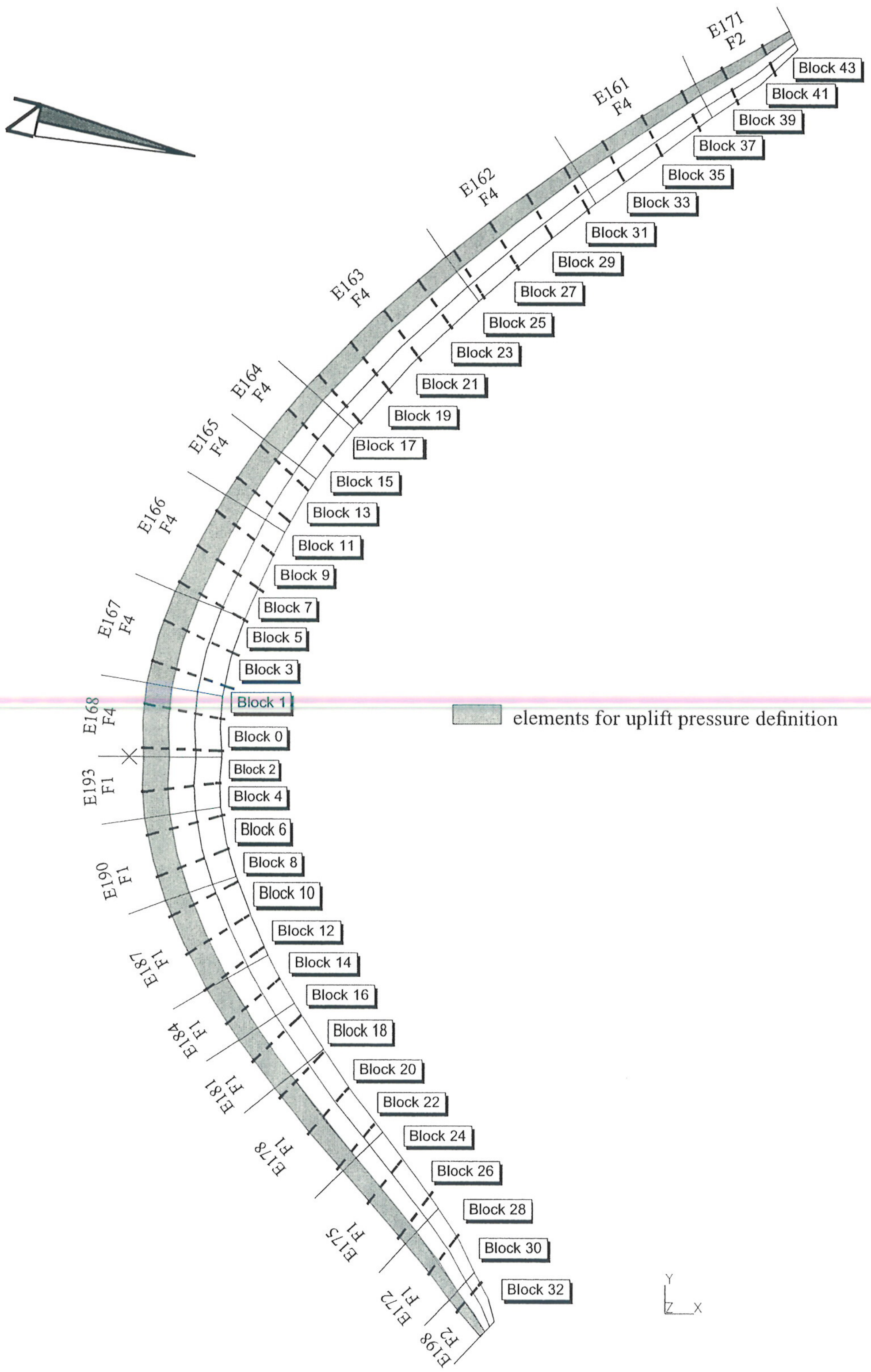

Figure 9: Element Numbers and Faces for Uplift Definition

2.2.2 System Requirements for FE-calculation

The FE calculation (for example with ABAQUS) needed:

255MB .... Scratch file sizes

10MB .... Result file (one step only)

200MB .... Memory

3 RESULTS

3.1 Required Output

The requested results are

- The formula and the parameters governing the radial crest displacements in terms of water level (actual and history) and temperature and
- A prognosis of the radial crest displacements for the period 1999 to 2000

The formula for the crest displacement as a function of water level and temperature can be of any type, e.g. polynomial. A separation of the contribution of the water load and of the temperature variation on the crest displacement should be possible.
The prognosticated values for the crest displacement for the period 1999 to 2000 should be provided in the same format as the values given.

S. Bonelli, H. F. Felix (will not be presented by the authors)

Delayed response analysis of temperature effect

DELAYED RESPONSE ANALYSIS OF TEMPERATURE EFFECT

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ABSTRACT: Long series of monitoring data are obtained during the routine operation of a dam. The traditionnal approach for performing data analysis is the linear statistical regression. This paper presents the application of auto regressive modelling techniques for performing delay analysis on air temperature measurements. The results are compared to seasonal analysis and direct analysis of concrete temperature measurements. There was no real difference between the results of the three models concerning the analysis of the measurements for the radial crest displacement of the Schlegeis arch dam.

1 INTRODUCTION

For monitoring data analysis, three families of deterministic models are commonly used: 1) structural models as finite-element models, non-structural models as statistical or neural networks models and hybrid models (Bossoney, 1994). Quantitative methods of analysis based on statistical models have been used on dam monitoring data for a long time, as reported in particular at ICOLD Congresses.

2 MODELLING

2.1 The seasonal model

The Hydrostatic-Season-Time (HST) method early proposed by Electricité de France for analysing pendulums in 1967 (Willm & Beaujoint) has proved to be a powerful tool for interpreting the behaviour of concrete dams in particular (ICOLD 1985, 1989, 2000). The mechanical behaviour is supposed to be linear elastic. The effect H of the reservoir level Z is given by a fourth-degree polynomial

$$H(Z)=a_1 w+a_2 w^2+a_3 w^3+a_4 w^4 \ , \ w(Z)=\frac{Z-Z_{min}}{Z_{max}-Z_{min}} \quad (1)$$

where Z_{min} is the minimum water level, Z_{max} is the maximum water level, $Z=Z(t)$ is the water level at time t, and $(a_{1..4})$ are four parameters. The seasonal effect S is represented by the sum of sine functions with one-year and six-month periods

$$S(t)=b_1\sin(\omega t)+b_2\cos(\omega t)+b_3\sin^2(\omega t)+b_4\sin(\omega t)\cos(\omega t)\ ,\ \omega=\frac{2\pi}{365} \quad (2)$$

where time t is given in days, and $(b_{1..4})$ are four parameters. The irreversible effect T (time drift), which is expressed in terms of monotonic time functions, is supposed to be negligible in the present study. The nine parameters $(c,a_1,a_2,a_3,a_4,b_1,b_2,b_3,b_4)$ seasonal model expression at time t_n is

$$Y_n=c+H_n+S_n \quad (3)$$

where $Y_n=Y(t_n)$, $H_n=H(Z(t_n))$ and $S_n=S(t_n)$.

2.2 The linear model

The thermal displacements of a thermoelastic beam depends on the thermal force, the thermal moment and some others functions integrating the temperature distribution in the cross-sectional area (Thornton, 1996). It is then possible to assume that the temperature effect Π is a linear combination of the concrete temperatures (Silva Gomes and Silva Matos, 1985):

$$\Pi(t)=d_1\theta^{12UP}(t)+d_2\theta^{12MI}(t)+d_3\theta^{12DO}(t)+d_4\theta^{15UP}(t)+d_5\theta^{15MI}(t)+d_6\theta^{15DO}(t) \quad (4)$$

where $(d_{1..6})$ are six parameters, and θ^{LC} is the concrete temperature measured at location LC. This approach can be considered as a spatial integration of the concrete temperatures. The eleven parameters $(c,a_1,a_2,a_3,a_4,d_1,d_2,d_3,d_4,d_5,d_6)$ linear model at time t_n is

$$Y_n=c+H_n+\Pi_n \quad (5)$$

where $Y_n=Y(t_n)$, $H_n=H(Z(t_n))$ and $\Pi_n=\Pi(t_n)$.

2.3 The ARMA model

The delayed effect Θ of the temperature is supposed to be proportional to the convolution of the impulse response of the dam structure and the ambiant temperature θ^{air}. Approximating the impulse response by taking an order one system defined by a characteristic time τ (Bonelli and Royet, 2001) leads to

$$\Theta(t)=\alpha_\theta\theta(t)\ ,\ \theta(t)=\int_0^t \theta^{air}(t')e^{-(t-t')/\tau}dt' \quad (6)$$

or equivalently

$$\frac{\theta}{t}(t)=\frac{\theta^{air}(t)-\theta(t)}{\tau} \quad (7)$$

As the data are provided every day, the semi-implicit Euler method can be used for discret expression, leading to the following autoregressive and moving average (ARMA) model (Box and Jenkins, 1976)

$$\theta_n=m_1\theta_n^{air}+m_2\theta_{n-1}^{air}+m_3\theta_{n-1} \quad (8)$$

$$m_1=\frac{(1-\beta)\delta t}{\tau+\beta\delta t}, \quad m_2=\frac{\beta\delta t}{\tau+\beta\delta t}, \quad m_3=1-\frac{\delta t}{\tau+\beta\delta t} \quad (9)$$

where δt is the time step, $0\leq\beta\leq1$ is the coefficient of the semi-implicit scheme and τ is a mean diffusion caracteristic time (unknown). This approach can be considered as a temporal integration of the air temperature. The seven parameters $(c,a_1,a_2,a_3,a_4,\alpha_\theta,\tau)$ ARMA model at time t_n is

$$Y_n=c+H_n+\Theta_n \quad (10)$$

where $Y_n=Y(t_n)$, $H_n=H(Z(t_n))$ and $\Theta_n=\alpha_\theta\theta_n$.

3. RESULTS

In the study of the displacement of concrete dams submitted to water level fluctuations and to temperature oscillations of the environnment (air, solar radiation and water), the behaviour is usually supposed to be linear (thermoelasticity) in a first approach. However, uncoupling the water level effect and the temperature effect is not possible because the temperature of the upstream face depends on the temperature of the water and on the level of the water. The superposition of water level and temperature effects can be assumed if water level effect means mechanical and thermal water level effect. If the operation is almost seasonal, another difficulty arises as consequence of the large correlation between the thermal and hydrostatic variables : this is the case in the present analysis, and the independance of these two effects can not be assumed.

The values of the parameters are defined to minimize the sum of the least square between the measurements y and the computed values Y.

The results are given table 1. The values of (Z_{min},Z_{max}) were chosen to be (1660,1782). For the ARMA model, to compute the temporal convolution (6), the first hundred days were excluded of the analysis.

The water level effect is nearly the same (figure 1), except for low water level (Z<1700 m). There was no real difference between the results of the three models for the total displacement during the period of analysis (figures 2,4,6) and the forecasting period (figure 8).

In the seasonal model, the thermal effect is assumed to be a function of the date of the year only, in other words to the season. Only the water level is required, but this model admits that the thermal conditions of the dam should be the same on the same date of the year. Despite this restrictive assumption, the results are quite good (figure 2). If the seasonal effect is supposed to be the temperature effect, it appears to have a one-year period only (figure 3).

The linear model and the ARMA models provide good results (figures 4 and 6), even if some peaks of temperature effects are not well explained (figures 5 and 7).

The ARMA models are widely used in many different fields, for example in financial forecasting (Clements and Hendry, 1998). These models can be the discret expression of a transfert function model (Young, 1998). Their application in the Civil Engineering context has been limited to date.

Model	Seasonal		Linear		ARMA
Date min	1		1		100
Date max	2557		2557		2557
N	2557		2557		2458
R^2	0.99		0.99		0.98
c	28.31		25.89		25.36
H					
a_1	-43.35		-72.59		-11.86
a_2	139.77		265.38		107.61
a_3	-137.38		-317.34		-142.34
a_4	90.86		174.78		105.13
S		Π		Θ	
b_1	8.90	d_1	-0.21	α_θ	-1.64
b_2	7.49	d_2	-0.67	τ	32.50
b_3	0.24	d_3	-0.58	(beta=0.5)	
b_4	-0.37	d_4	-0.52	(dt=1)	
		d_5	3.79		
		d_6	-1.43		

Table 1. Results of analysis, value of coefficients of the models.

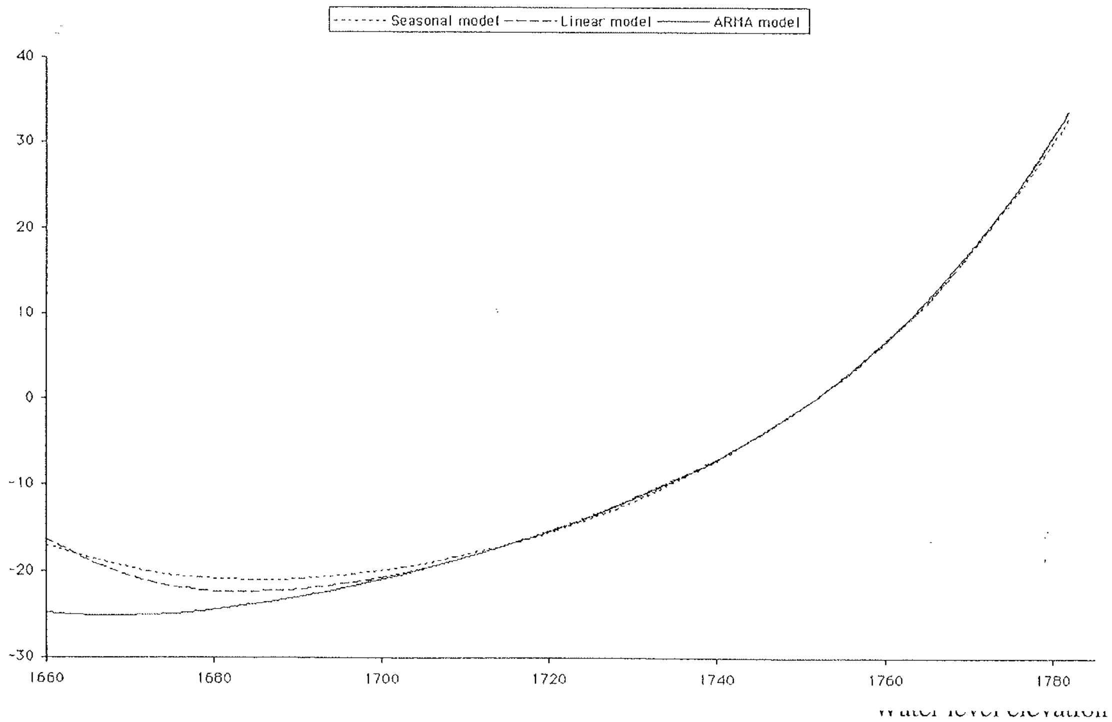

Figure 1. Water level effect H(Z), comparison between seasonal, linear and ARMA models

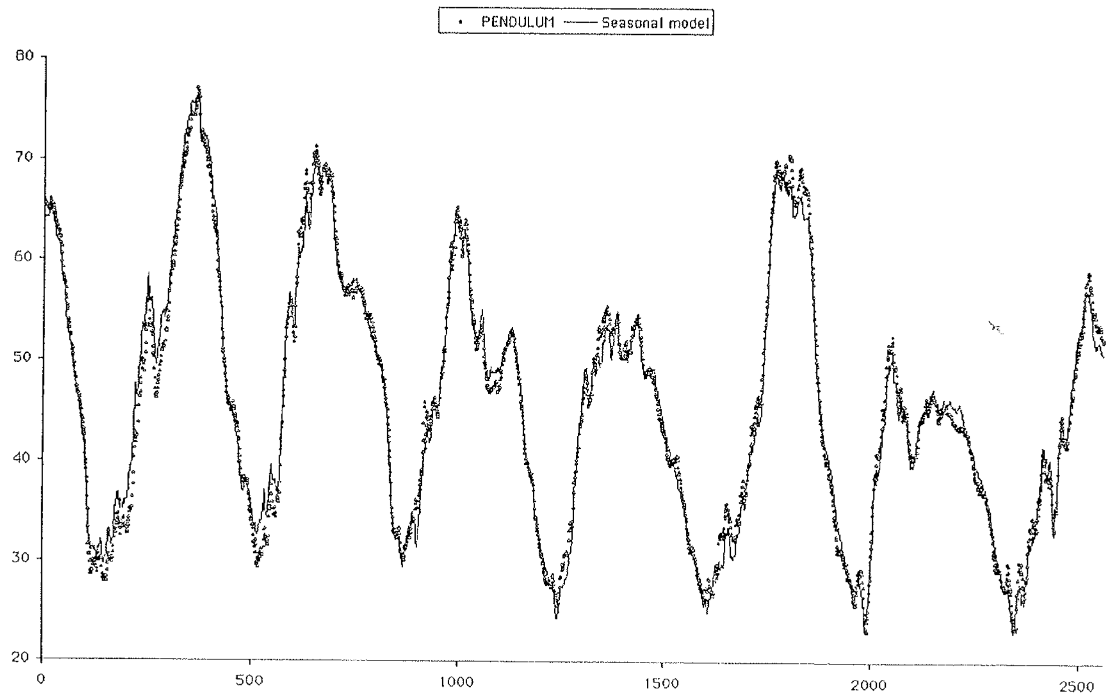

Figure 2. Radial displacement, comparison between measurements y (pendulum) and seasonal model Y=c+H+S

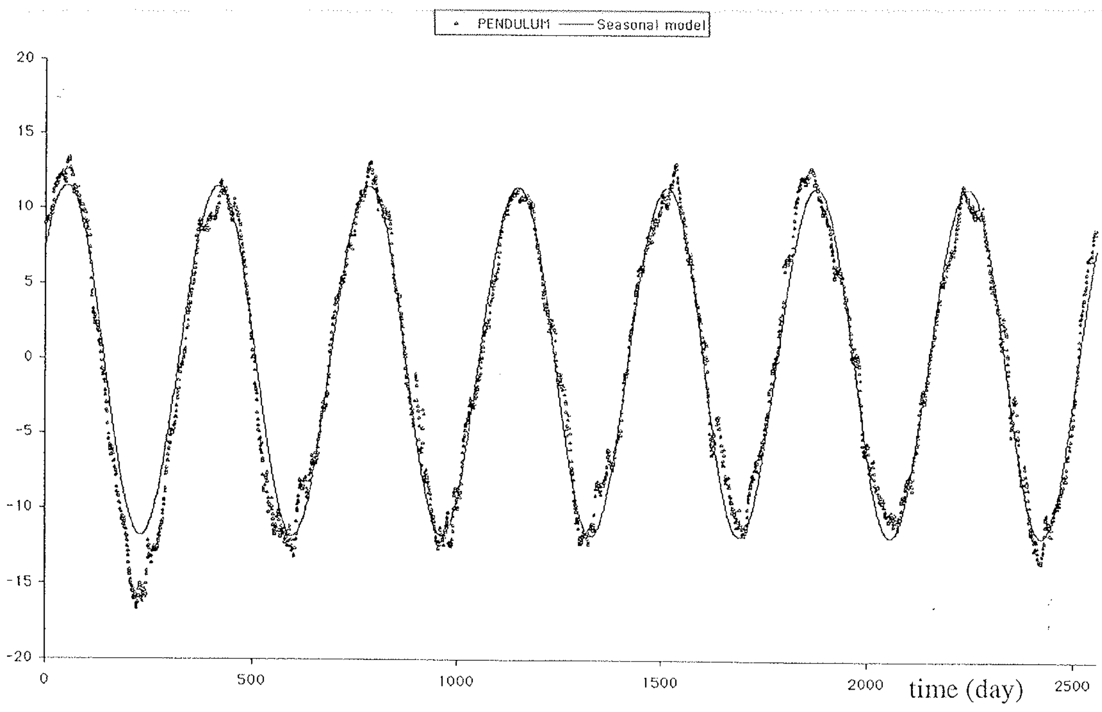

Figure 3. Temperature effect, comparison between measurements y-H at constant water level (pendulum) and seasonal model S

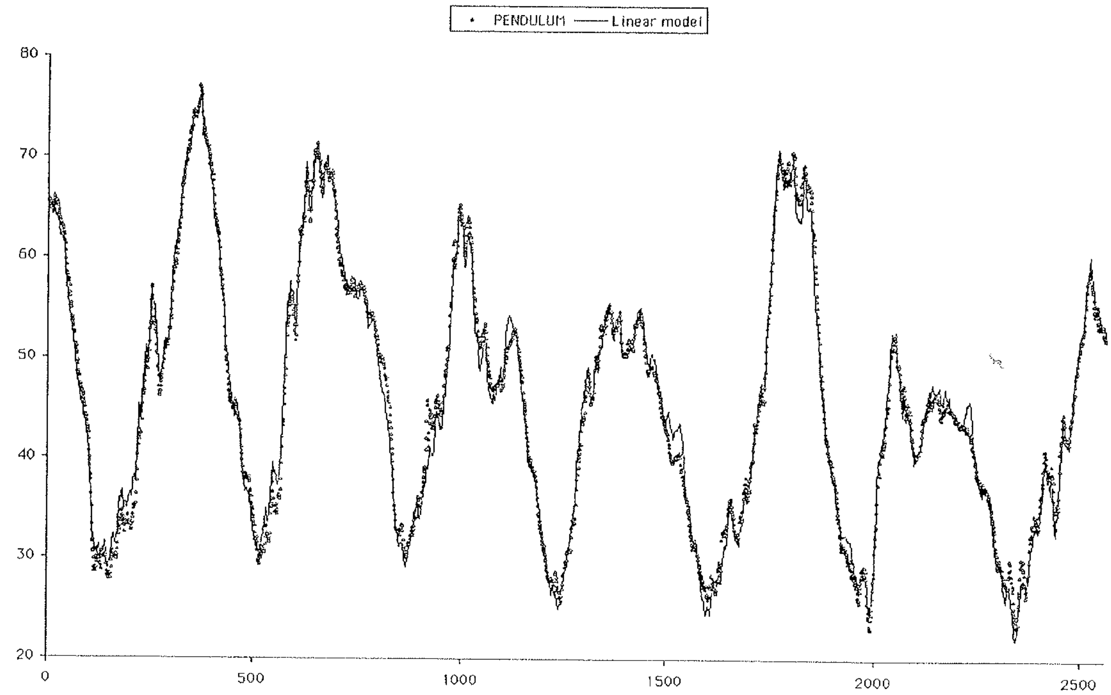

Figure 4. Radial displacement, comparison between measurements y (pendulum) and linear model $Y=c+H+\Pi$

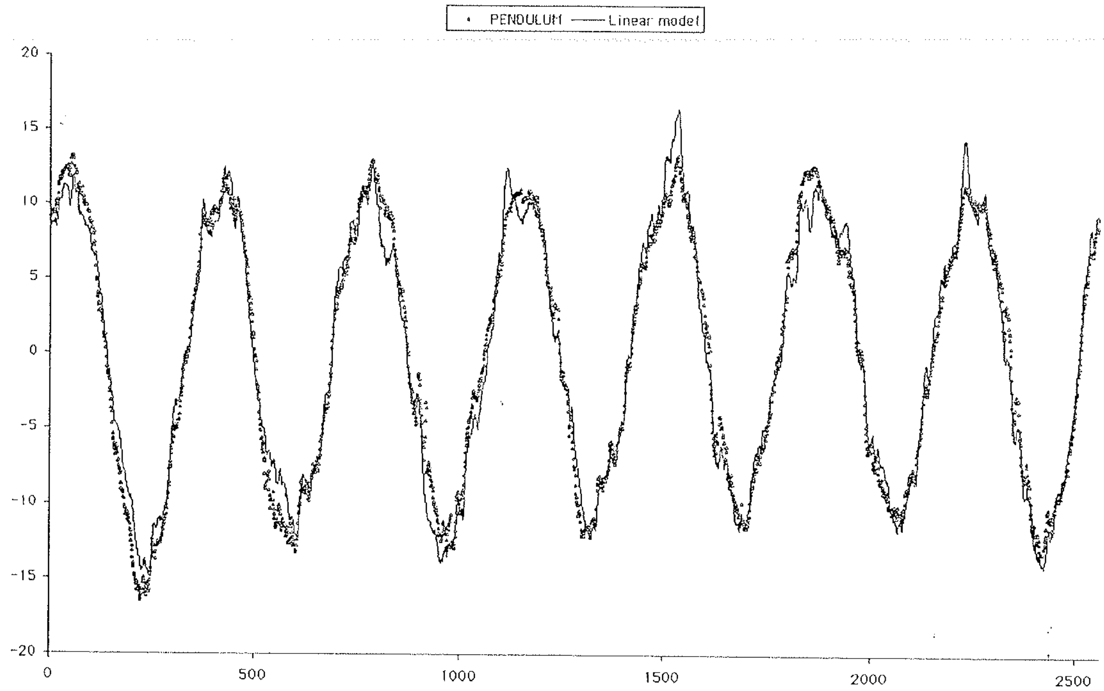

Figure 5. Temperature effect, comparison between measurements y-H at constant water level (pendulum) and linear model Π

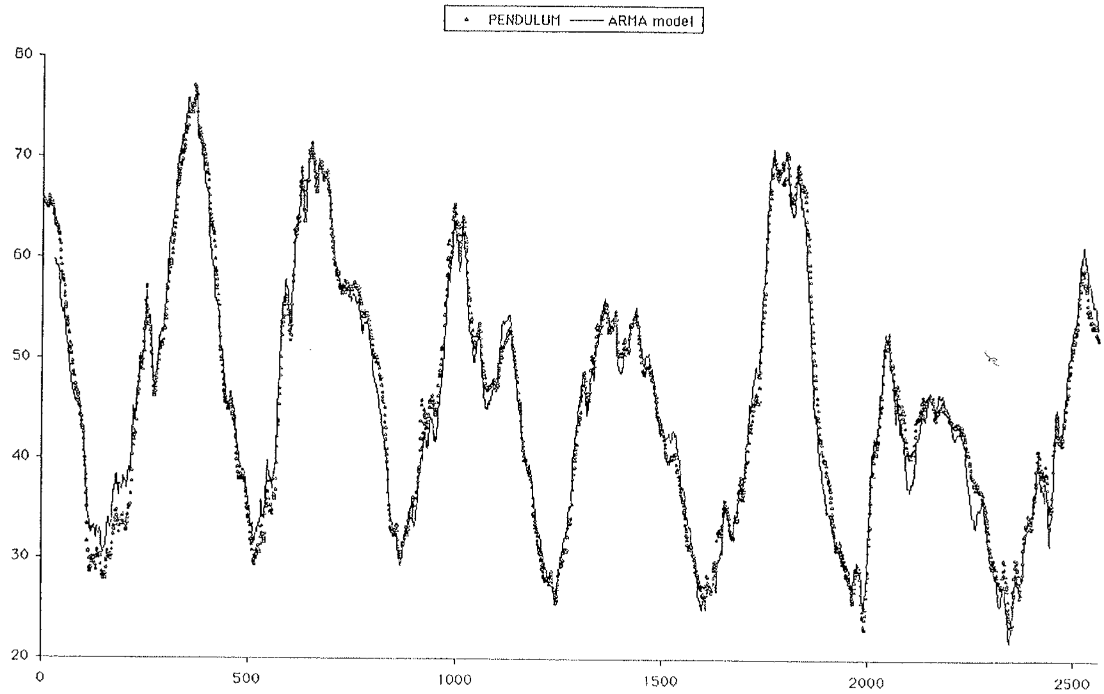

Figure 6. Radial displacement, comparison between measurements y (pendulum) and ARMA model Y=c+H+Θ

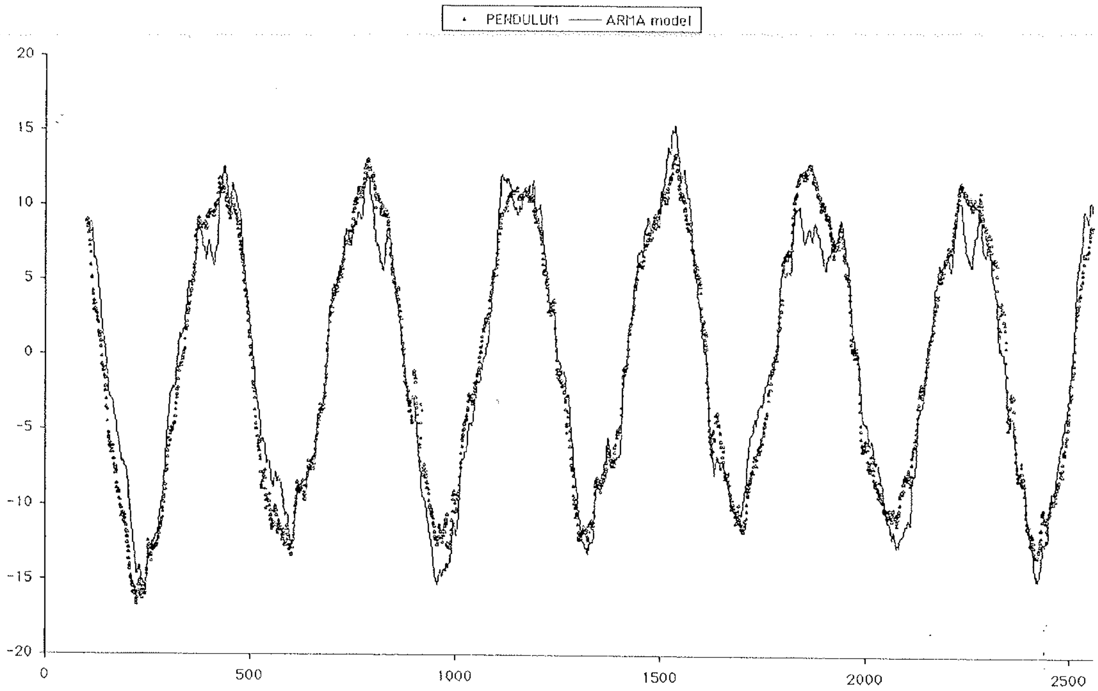

Figure 7. Temperature effect, comparison between measurements y-H at constant water level (pendulum) and ARMA model Θ

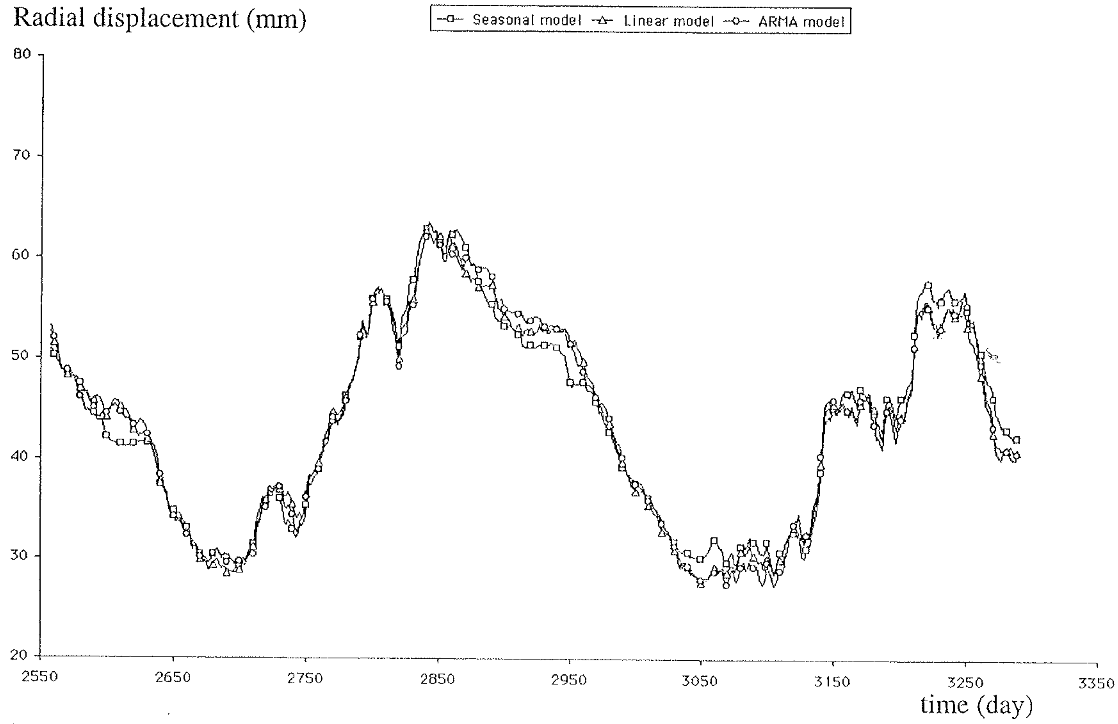

Figure 8. Radial displacement, forecasting for the period 1999 to 2000, comparison between seasonal, linear and ARMA models

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Interpretation of an arch dam behaviour using enhanced statistical models

INTERPRETATION OF AN ARCH DAM BEHAVIOUR USING ENHANCED STATISTICAL MODELS

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ABSTRACT

The readings of a virtual unique pendulum representing the crest deflection of SCHLEGEISS arch dam have been analysed over the 1992-1998 period, in order to predict deflections which occurred in the following two years. The Hydrostatic–Seasonal-Time (HST) statistical method has been preferred to structural analyses because all conditions were optimal for its use (large experimental period, few non-cyclic delayed effects, no major non-linear behaviour expected). The HST model was prepared with an experimental version of the CONDOR software, developped by Coyne et Bellier. It provided a prediction of the deflection at crest with an expected average accuracy of 1.5 mm. It also allowed a detailed analysis of time effects to be carried out, during which a significant step of the pendulum readings has been identified to have occurred in summer of 1993. This might allow a realistic structural model (MODAP) to be used as the next step to explain this special event.

Key words: Safety assessment, Concrete Dam Structures, Arch Dam, Monitoring, Interpretation, Statistics

1 INTRODUCTION

The objective of the present exercise is to predict movements of SCHLEGEISS arch dam over the period from 1999 to 2000, starting from observations accumulated during the previous period 1992-1998. For such purpose, two main methods are applicable: the structural, so-called 'deterministic' method, and the statistical one. Any combination of both is also possible (Carrère [1]). Conditions prevailing for the subject are actually especially favourable to statistical analyses, since delayed effects are likely to be small. For this reason, the Authors decided to found their contribution mainly upon statistical analyses, referring to a very simple structural model only for verification.

The statistical analyses have been carried out with the help of CONDORpc, an experimental software especially developed for the interpretation of dam monitoring results. CONDORpc has two working modes:

- Spatial analysis, where readings from several instruments throughout the structure are correlated ; the Principal Components Analysis (PCA) algorithms are used; the "pc" in the name of the software stands for reference to this method;

- Temporal analysis, giving relations between readings at different moments for the same instrument; this refers to the "Hydrostatic/Seasonal/Time (HST)" method, with some enhancements already implemented and experienced in the commercial version CONDOR II (Carrère [1], Boutayeb [2]).

Both methods have been used. It has been found that the temporal analyses provide more accurate predictions, most likely because Theme C problem refers to only one pendulum, also because the parameters leading the dam movements (i.e. hydrostatic load, temperature and rheology of materials) are well represented by the HST functions.

2 PREPARATION OF THE STATISTICAL ANALYSIS

A database dedicated to CONDORpc has been prepared with the main characteristics of the dam (crest level, height, date of impoundment) and with the readings extracted from the file MONIT.TXT provided by the Formulator. Statistical models have then been prepared for secondary readings (temperatures and water level), with the exception of temperature T15mi, which has a standard variation of 0,1°C only, and is therefore nearly not significant. The attention has been called upon three special aspects, which are discussed below.

2.1 Seasonal variations of the reservoir level

The reservoir level follows rather well a cyclic yearly variation (Figure 1), however the coupling with the season is imperfect with an explanation coefficient of only 0.57.

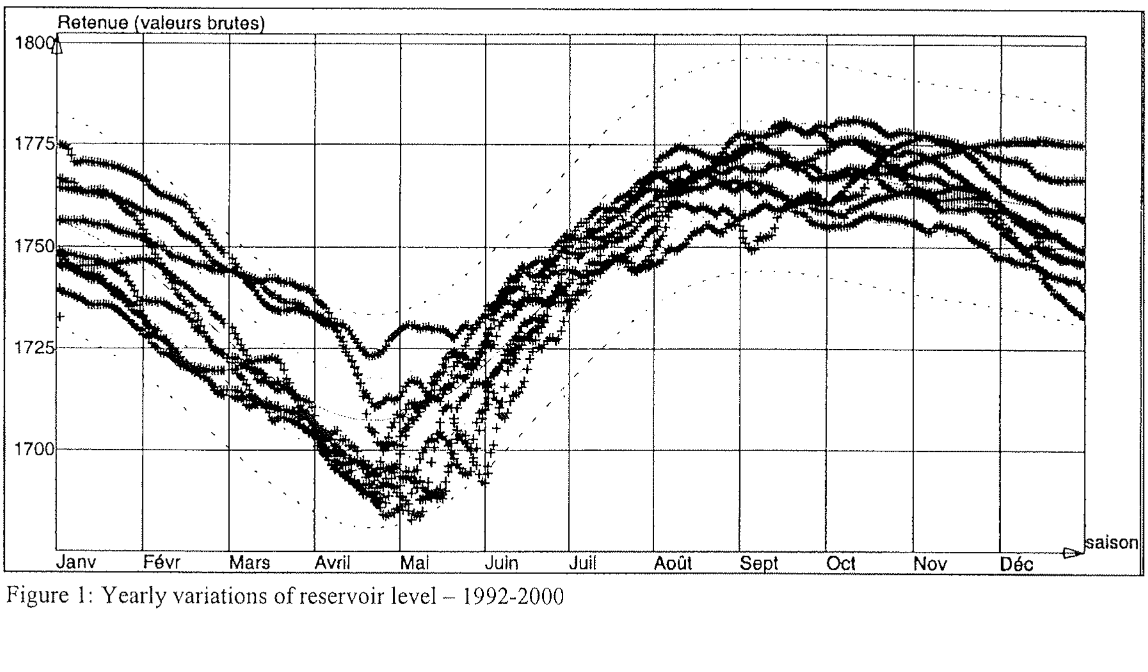

Figure 1: Yearly variations of reservoir level – 1992-2000

A rather good independence of hydrostatic ("H") and seasonal ("S") functions may therefore be expected which is a favourable factor.

2.2 Seasonal variations of temperature readings

Temperatures measured by instruments located close to the downstream face of the dam follow rather well the air temperature. Temperature T12up is measured close to the upstream dam face at el. 1750.65; it is obviously influenced by the variations of the reservoir level in front of it, as shown by Figure 2. Readings on other instruments follow rather well seasonal cycles, especially T12mi which is likely to influence mostly the pendulum readings variations (Figure 3).

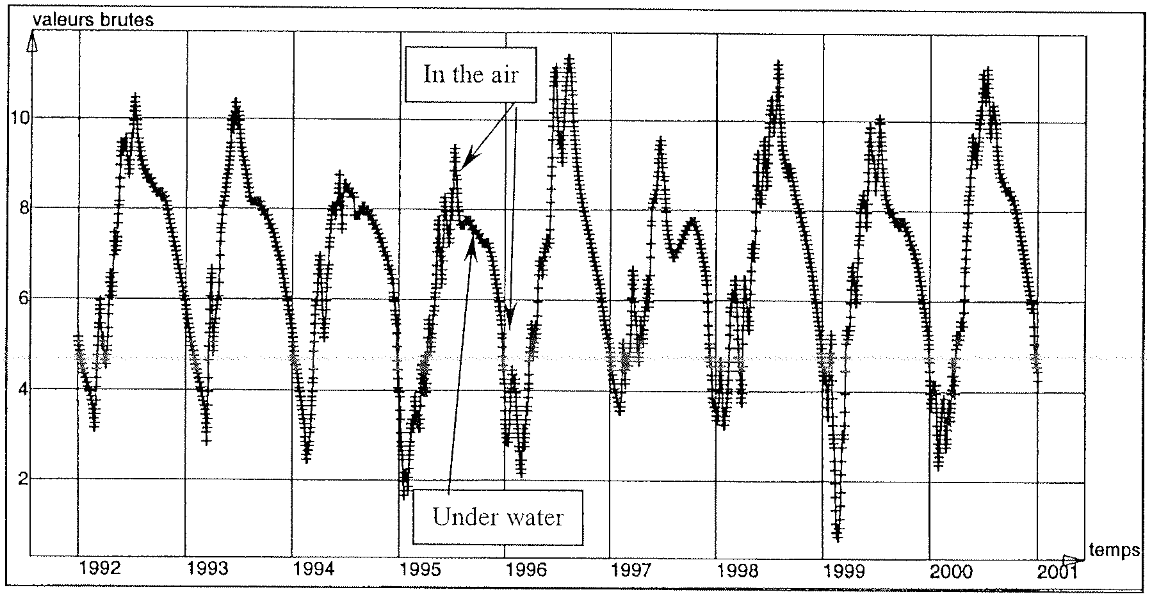

Figure 2: Variations of T12up Vs time – 1992-2000

2.3 Abnormal readings

The model for temperature T12mi has detected an important anomaly, which occurred between 25/06 and 26/07/94 inclusive. This can be clearly seen on Figure 3. The lower graph presents readings corrected from hydrostatic and seasonal effects Vs time, it shows that the quick raising is about 3°C in 5 days, and it is neutralised during the next 25 days.

The explanation of this phenomenon has not been identified. It may be due to a local change of thermal conditions e.g. the isolation system being removed during a few days, or something similar. Whatever the reason, it has been assumed that the phenomenon was only local and has not influenced significantly the whole structure. Readings during the abnormal period have therefore been discarded from further analyses where temperatures were considered.

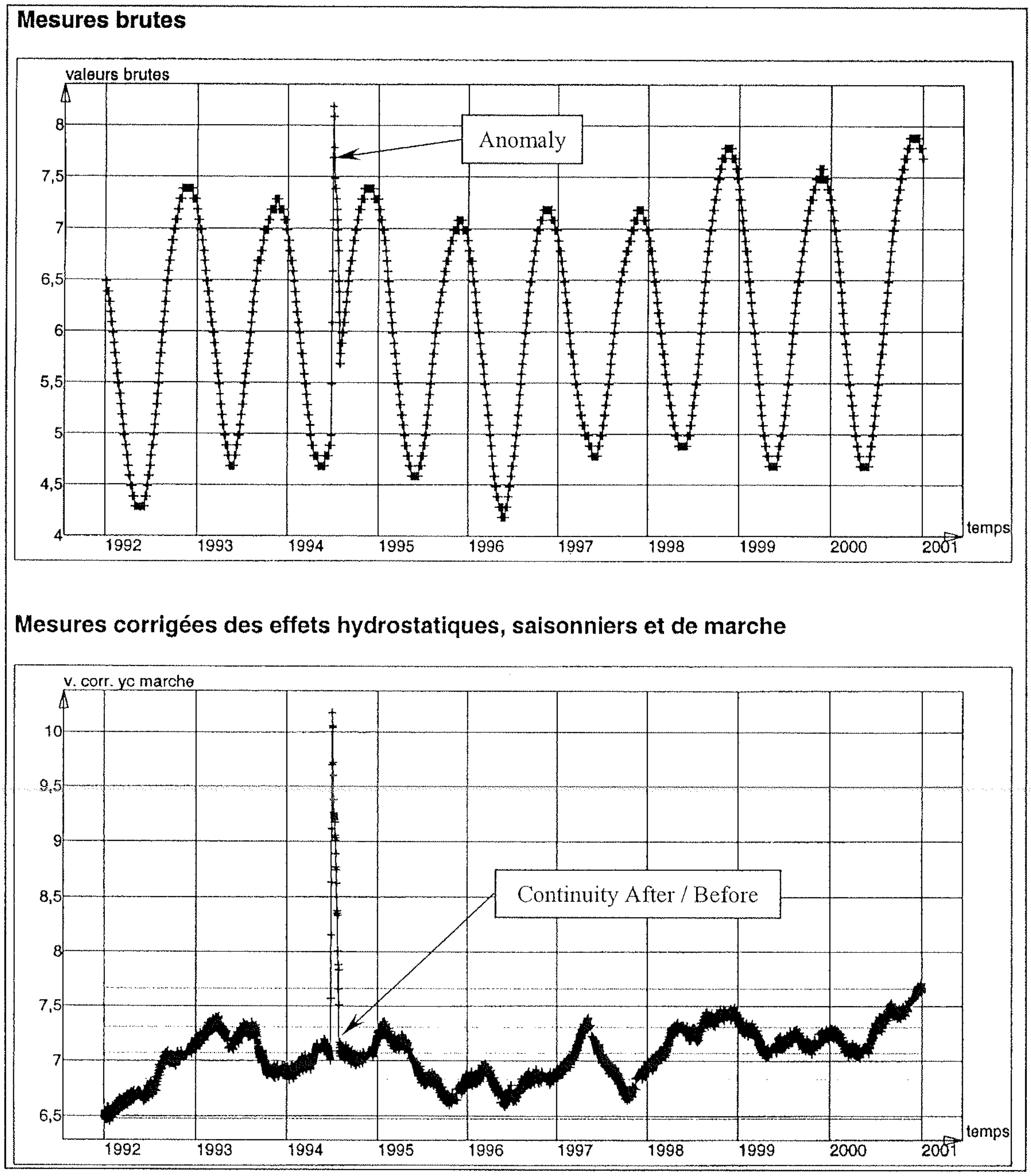

Figure 3: Fac-Simile of CONDORpc graph for T12mi instrument: Variations of direct and corrected values Vs time

3 PREPARATION OF THE HST MODEL

Several models have been prepared successively, until reaching a prediction considered optimal. Results have been evaluated on the basis of the residual variation coefficient, which quantifies the average difference between the prediction and the measure. All models are based on the 2557 observations available between 01/01/1992 and 31/12/1998.

3.1 Standard model

The default model automatically calculated by the software has discarded insignificant functions and has kept two "H" functions (Z, Z^2), all four seasonal "S" functions (SinS, CosS, Sin^2S and SinS*CosS), the time-drift function T, and the drift reduction function Exp.(-T). The residual standard deviation is 1.5 mm which corresponds to an explanation coefficient as high as 87.9 percent (Figure 4).

3.2 Model without time drift reduction function

It has been considered that the drift reduction could not be considered a significant phenomenon, more than 20 years after the first loading of the dam. The drift reduction function Exp.(-T) has therefore been discarded from the second model. In this one, the residual standard deviation and the explanation coefficient are nearly unchanged from the previous. Figure 5 shows that the drift through the period is increased to 0.4 mm/year, instead of 0.25 mm/year before.

3.3 Model with a step function

When examining the corrected values Vs time given by the second model (Figure 5 above), the drift appears to be mainly due to a sudden variation during the year 1993. A third model has therefore been prepared, with incorporation of a step function. A step function is a special feature implemented into CONDORpc, which is equal to 0 before a given date, and becomes equal to 1 from this date onwards. After optimisation, the step has been located to the date of 05/08/1993 and it has shown a step value of 2,3 mm towards downstream. This can be seen on Figure 6.

The main features of the model are shown on Figures 7 and 8, which are *fac-simile* of model-description sheets issued by CONDORpc:

- the residual variation coefficient is reduced to 1.36 mm,
- the corresponding explanation coefficient is higher than 89 percent, which is an excellent result,
- the influence of the seasonal effect is 27 mm, with the maximum at the end of August and the minimum at the beginning of March,
- the influence of the hydrostatic effect is 57 mm when the water level varies from el. 1680 to el. 1782 (the range of experience is limited downwards to el.1680 and the correction function is no more valid below),
- the drift is now only 0,09 mm/year, much less than in previous models and its significance is low.

The physical significance of the step in 1993 is discussed further below.

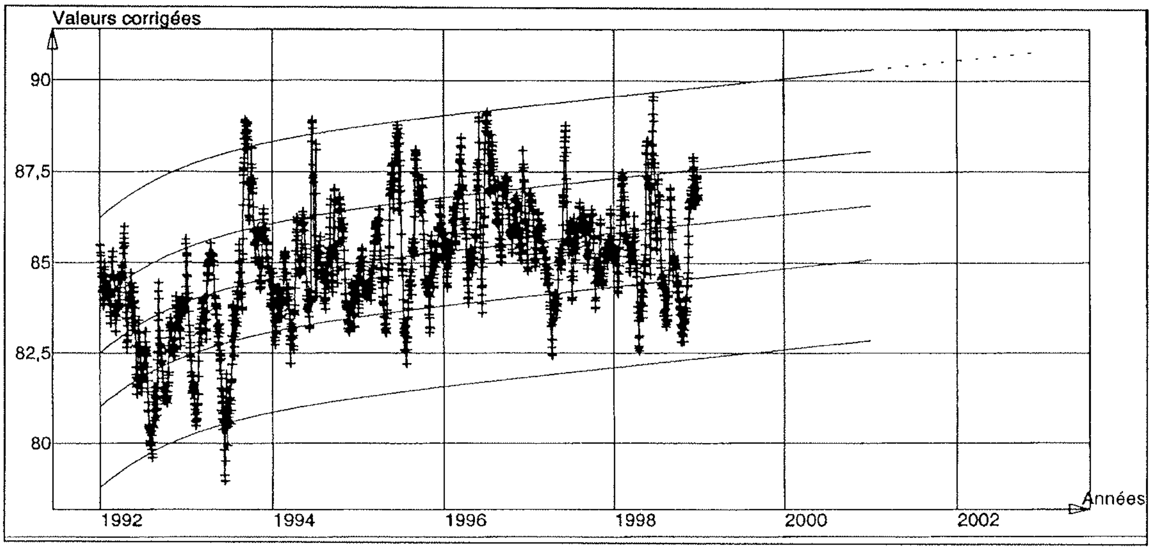

Figure 4: Standard model for the pendulum automatically prepared by CONDORpc – Std. Dev.=1.49 mm

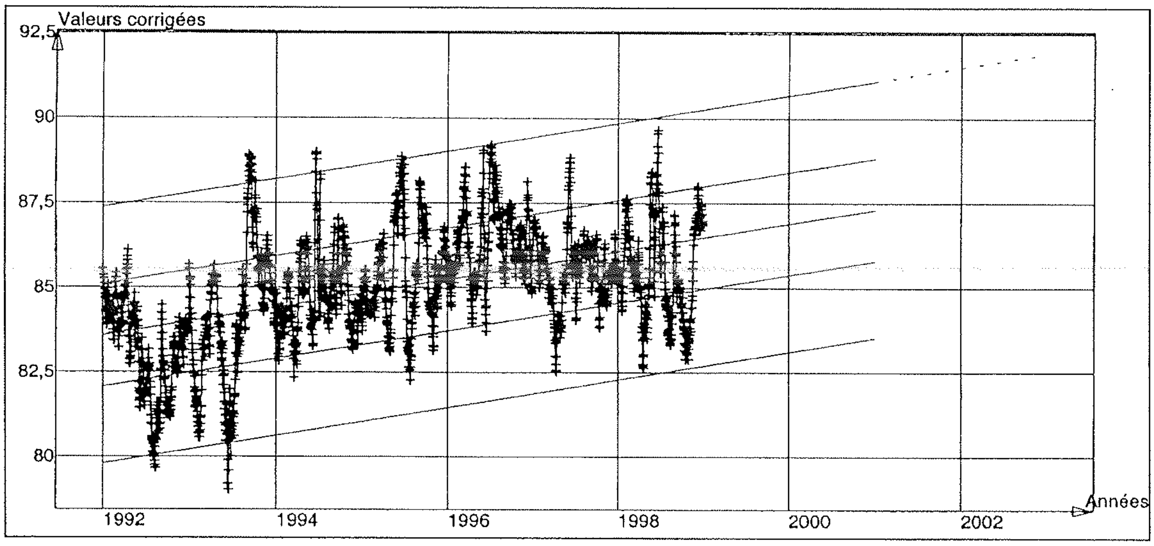

Figure 5: Model for the pendulum without reduction drift function – Std. Dev.=1.51 mm

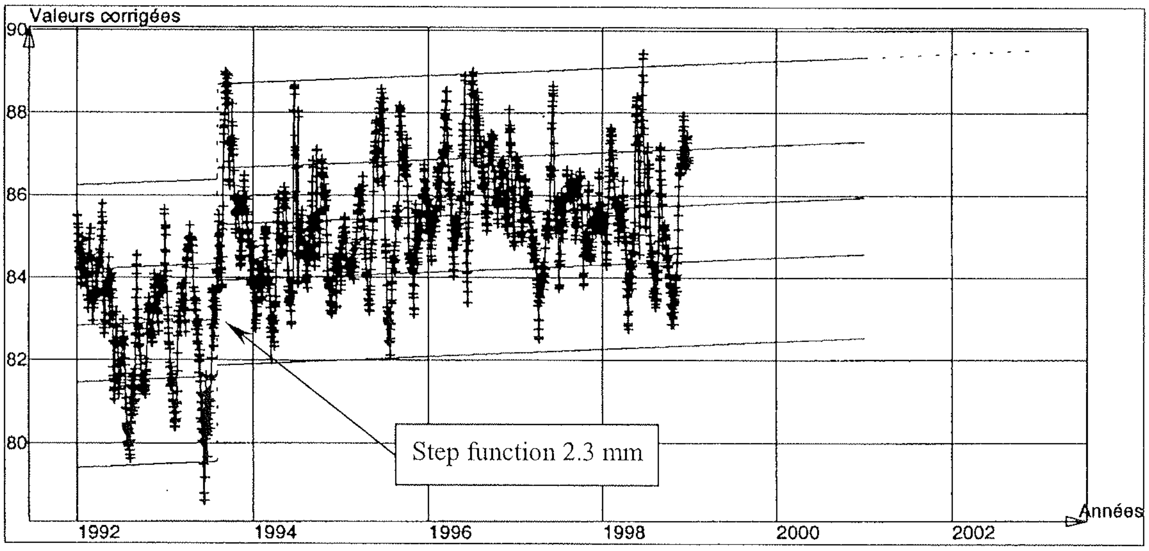

Figure 6: Model for the pendulum with a 2.3 mm step function in August 1993 – Std. Dev.=1.36 mm

Any attempt to obtain better adjustment of the model on readings over the 1992-1998 period have failed, which means that the model described above is the optimal one. It has therefore been selected to carry out the prediction over the 1999-2000 period.

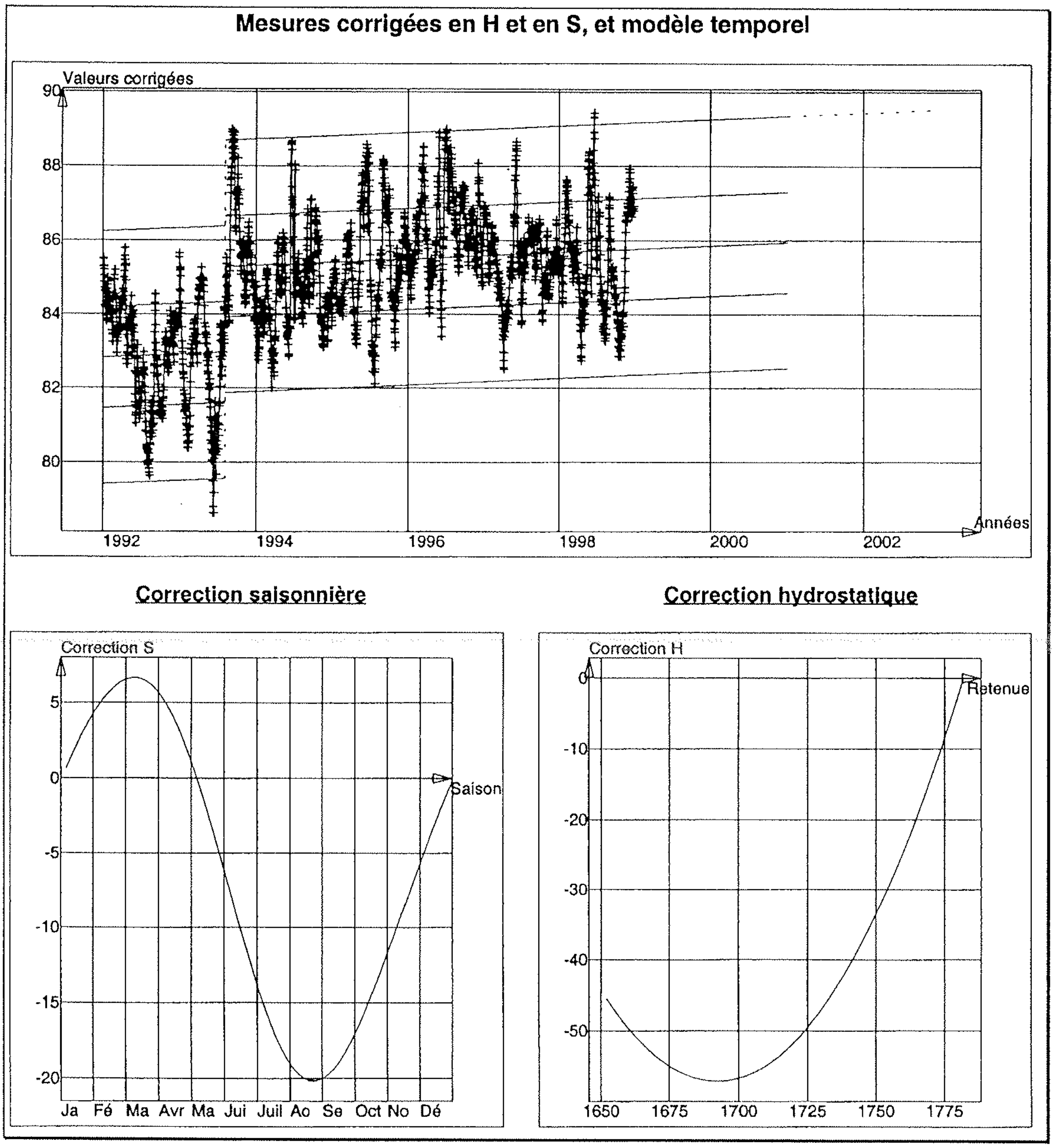

Figure 7: Fac-Simile of model-description sheet prepared by CONDORpc for the pendulum – Correction functions

Top: readings corrected with seasonal and hydrostatic effects

Left: seasonal effect function

Right: hydrostatic effect function

Grandeur Pendule (mm, Virtual unique crest pendulum at)

Modèle HST actif du 31/07/2001 16:43:05

Données

Nombre de séries de mesures valides utilisées : 2557
Date de la première série : 01/01/1992
Date de la dernière série : 31/12/1998
Durée de la période : 7,00 années

Résultats statistiques

Ecart-type des mesures brutes:	EctB= 12,496 mm
Ecart-type des écarts avec le modèle:	EctC= 1,365 mm
Coefficient global de corrélation:	CgC = 0,994
Coefficient global d'explication:	**CgE = 89,1 %**

Formulation du modèle

					Coefficients de Student-Fisher
Pendule (mm)		=	82,8254		
+	Z	*	-165,5969 }	*correction*	204,700
+	Z²	*	120,0897 }	*selon la*	136,300
+	Z3	*	}	*cote de*	
+	Z4	*	}	*retenue*	
+	(1-CosS)	*	-7,0327 }	*correction*	152,900
+	SinS	*	11,2697 }	*selon*	104,700
+	Sin²S	*	1,3601 }	*la*	15,430
+	SinS.CosS	*	-1,5820 }	*saison*	17,300
+	T/Tbt	*	0,0893 }	*correction*	3,942
+	(T/Tbt)²	*	}	*selon le*	
+	Exp(-T/Tbt)	*	}	*long terme*	
+	{ D>=DDM }	*	2,3134 }	*terme de marche*	24,250

Constantes de la formulation

Zphe = 1782,000 Hbr = 130,000
Tbt = 1 (en années) Dinit = 01/01/1992
DDM = 05/08/1993

Notations

Z : creux relatif: (0 = plein, 1 = vide), calculé par la formule: Z=(Zphe-H)/Hbr ,avec
. Zphe : cote des plus hautes eaux, et
. Hbr : hauteur du barrage,
H : cote de la retenue lors de la mesure courante,
S : saison, comptée de 0 au 1er janvier à 360 au 1er janvier suivant,
T : temps en années à partir de la date initiale Dinit (correspondant ou non à la mise en eau),
Tbt : constante de temps,
DDM : date de marche à partir de laquelle la correction du terme "de marche" est appliquée,
D : date (et heure) de la mesure courante.

Figure 8: Fac-Simile of model-description sheet prepared by CONDORpc for the pendulum – Numerical description

4 RESULTS

Results are provided under the shape of

- the formulation of the model, which allows calculation of the predictive value for the pendulum reading at any moment,
- the predictive values for the 1999-2000 period.

Based on the residual variation coefficient of 1.36 mm obtained during the 1992-1998 observation period, the predictions given below are expected to have an average accuracy of ±1.5 mm.

4.1 Model formulation

The model is expressed as a function of

- the relative reservoir level Z=(MaxWL-WL)/Height = (1782-WL)/130,
- the season angle, rounded from 0 at January 1st to 360 at December 31st,
- the time T in years since January 1st, 1992.

The temperature values are not needed in the formulation.

The full formulation of the HST model is given in Table 1 below.

Prediction (mm) =	82.8254		Constant
-	165.5969	* Z	Hydrostatic functions
+	120.0897	* Z²	Z = (1782-water level) / 130
-	7.0327	* [1-Cos(S)]	
+	11.2696	* Sin(S)	Seasonal functions
+	1.3601	* Sin²(S)	S = 0 to 360 from 01/01 to 31/12
-	-1.5820	* Sin(S)*Cos(S)	
+	0.0893	* T	Time drift function (T in years from 01/01/1992)
+	2.3134	* {T>=Tstep}	Step function =1 if T>=(Tstep = 05/08/1993), otherwise =0

Table 1: Formulation of the statistical HST model for the pendulum

4.2 Predictive values for the pendulum

CONDORpc directly gives predictive values for individual instruments and series of readings. In order to provide the file of results under the specified format, the statistical model prepared by CONDORpc has been copied into an EXCEL sheet. Numerical predicted values for the pendulum for the period 1999-2000 are included into the BW6C_Cob1.TXT file. The same spreadsheet has been used to prepare Figure 9, where readings and predictions can be compared.

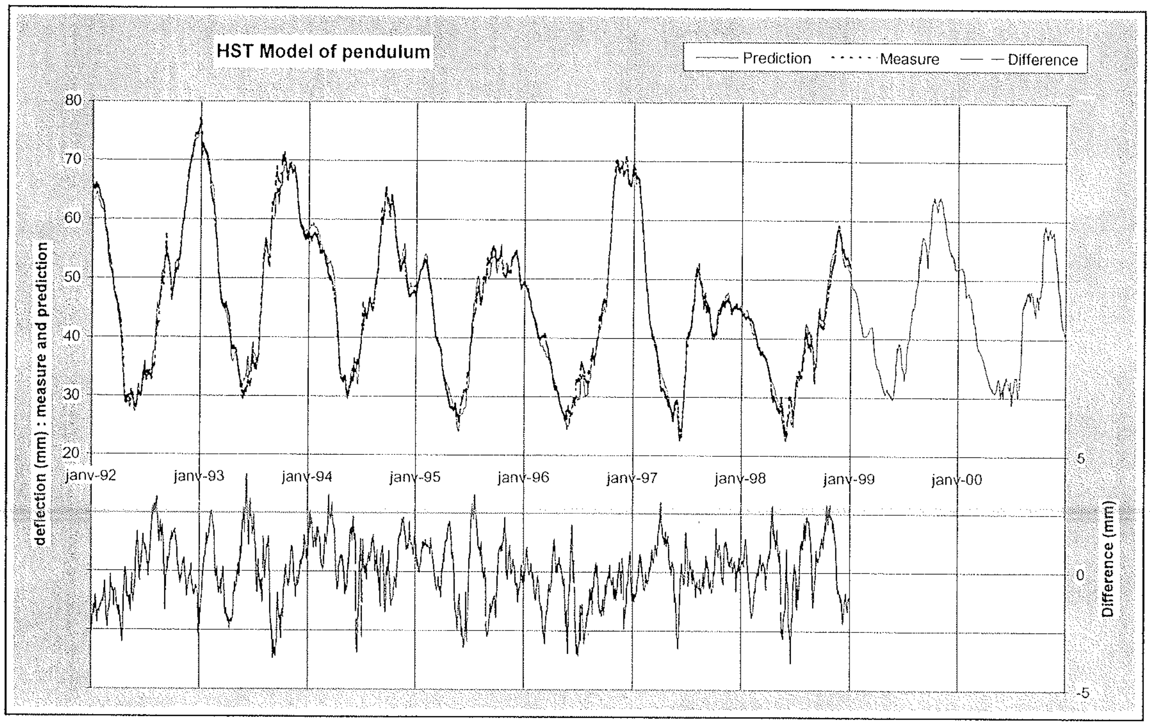

Figure 9: Comparison between readings and predicted values, by statistical HST method

5 DISCUSSION

5.1 Input of structural finite element analysis

In order to check the consistency of hydrostatic and seasonal functions included into the statistical model used, a simple FEM model of SCHLEGEISS arch dam has been prepared and used to calculate the crest deflections under simple hydrostatic and thermal effects. The COQEF3 code has been used (Hamon [3]). In this code which is devoted to the quick analysis of arch dams, the dam is modelled with thick shell elements and the foundation is represented with infinite VOGT elements. The mesh has been derived from the one provided by the Formulator. All suggested properties of the materials have been used (including concrete modulus Ec=25 GPa), with the exception of the foundation which has been idealised as an isotropic material, with a unique modulus of Er=20 GPa. The thermal load case has been strongly simplified, in that way that variations of temperature have been considered uniform from upstream to downstream; they have also been supposed to be synchronised and proportional to that measured at point T12mi, with the ratios shown in Table 2 below.

Elevation	1783.00	1764.50	**T12mi** 1750.65	1744.00	1724.00	1704.00	1685.00
Temperature variation (°C)	2.6	1.5	1.0	0.9	0.6	0.4	0.2

Table 2: Temperature distribution Vs elevation

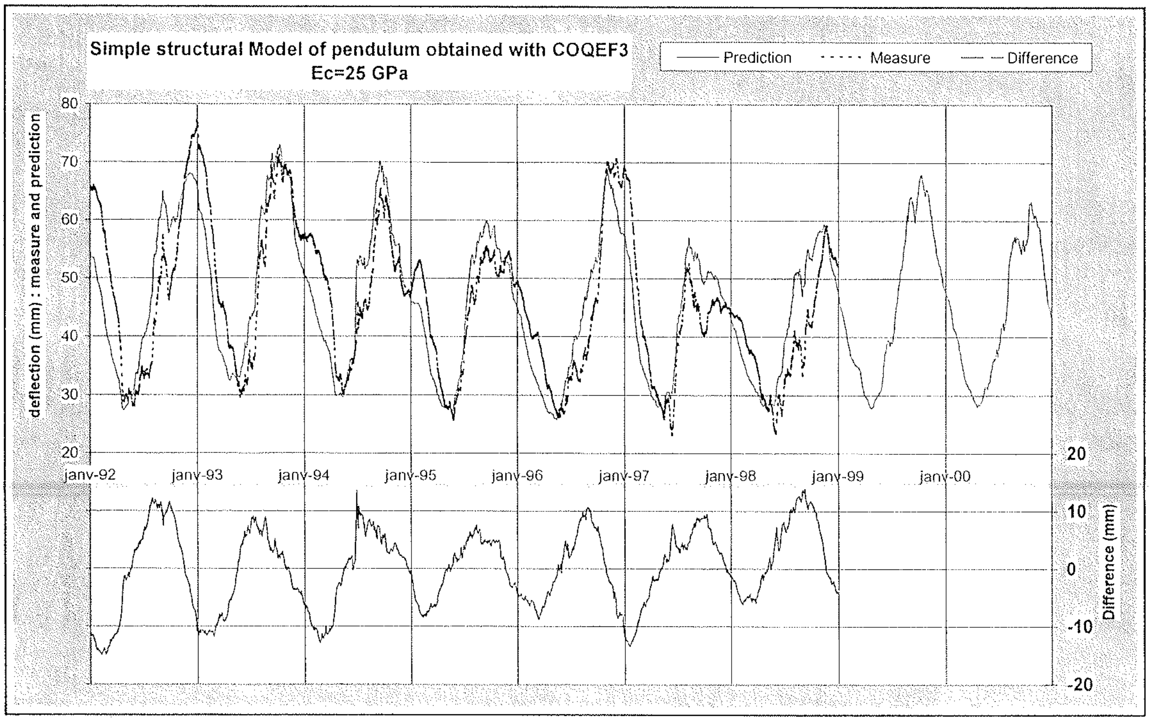

Figure 10: Comparison between readings and predicted values, by simple structural FE model

The adjustment obtained with this simple structural model is much poorer (Figure 10) than the one obtained with CONDORpc. The difference is approximately four times more (notice that the vertical difference scale on the right is different from Figure 9).

The hydrostatic effects on crest deflection given by both FE and HST models are presented on Figure 11. FE results are also presented for concrete moduli of 33 and 45 GPa (with a constant Ec/Er ratio). The comparison suggests that the equivalent elastic modulus is close to 25 GPa at low hydrostatic loads (say between 1700 and 1750), but surprisingly it seems to increase when the water level is high, up to 35 GPa or even more. Altogether it is suggested that the best unique modulus should be selected around 33 GPa, instead of 25 indicated by the Formulators.

As far as thermal effects are concerned, the deflection at crest level given by the FE model for this simplified load case is 3.3 mm only. Considering that the cyclic variation of T12mi is 2.7°C in average, the corresponding thermal component of the pendulum variations should be 8.9 mm. This is much less than the seasonal variation of the pendulum given by the statistical model, which is 27 mm i.e. 3 times higher. Such excessive discrepancy is likely to be due to the simplification of the thermal distribution assumed in the FE model load case: the bending effect due to temperature differences through the dam thickness are overlooked, as well as phase differences at different levels. Moreover, no temperature reading is provided close to the crest level, where the dam is less thick and where rapid temperature variations are likely to induce significant part of the crest deflection.

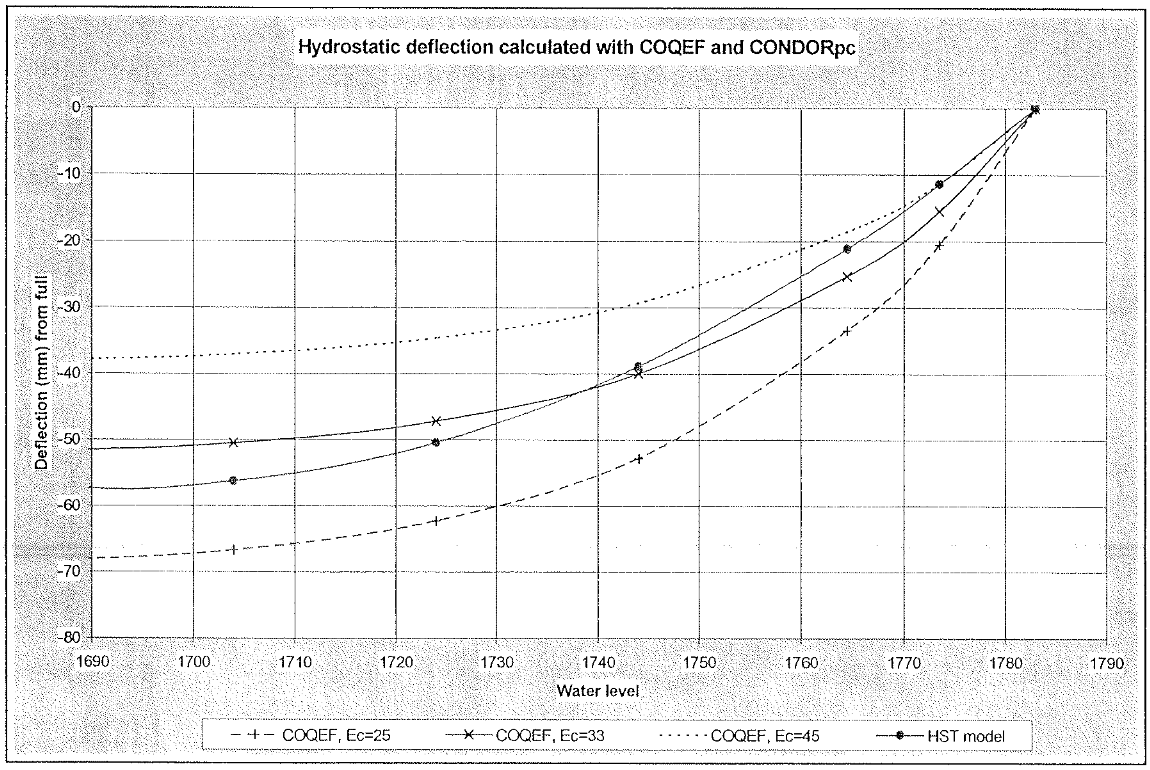

Figure 11: Comparison of hydrostatic effects on Crest deflection given by statistical HST method and by FE method

The simple FE model used here represents rather well hydrostatic effects but is not realistic at all for thermal aspects. To reach the same level of accuracy as that given by the HST model would require a time-history of the full temperature field in the whole dam to be determined as a preliminary, starting from the 6 instruments readings plus air temperature. This kind of exercise requires an amount of work which is out of scale with that spent for the statistical analysis.

5.2 Time effects

Time effects have been identified by all statistical models. They are relatively important as shown in Table 3. As already mentioned, most of the time effect is concentrated within a short period in August 1993. This has been idealised as a single step of 2.3 mm towards downstream. One may notice that this period corresponds to an exceptionally high water level above 1780: the maximum for the period was indeed at the beginning of October 1993, where el. 1781.52 was reached. It may be supposed that some adjustment of the dam structure (and most likely of its foundation) occurred under such unusual load conditions. The determination of the time step during the model optimisation is not very accurate and another value within 2 months more or less would not change other parameters. Once the step is taken into consideration, the remaining drift becomes nearly negligible.

Model n°	Drift / year	1992-1998 Drift	Step	Total time effect
1	0.25	3.6	0	3.6
2	0.41	2.9	0	2.9
final	0.09	0.6	2.3	2.9

Table 3: Time effect on the pendulum (mm) given by statistical models for the 1992-1998 period

5.3 Conclusion

Statistical models and so-called "deterministic" models can be used separately or together to interpret monitoring data.

Statistical models are often simpler and should therefore be preferred whenever possible. The most disseminated one is the HST method. However there are restrictions to its use, the main ones are:

- when data are not available over a rather long period of time and in different load conditions (i.e. start of first impounding),

- when strongly non-reversible effects are expected, to some extent,

- when delayed non-cyclical effects are observed (e.g. piezometric levels in seepage fields), although recent improvements of methods allow to take them into account.

The problem proposed for Theme C is ideal for use of the HST method. CONDORpc allowed analysing in detail the delayed effects, thanks to the step function implemented in it. The adjustment of the model with readings of the 1992-1998 period is rather good, and it is expected that its prediction for the 1999-2000 period will have an average accuracy around 1.5 mm.

The Principal Component Analysis (PCA) is another statistical method available to determine whether the behaviour of a dam is consistent with the past or not. Its principle consists in checking if the synchronism between different instruments is "as before" or not. However it is restricted in practice to conditions where correlations between different instruments are linear and reversible. It has been found that it was not the case in the frame of Theme C, however it might be of interest with consideration of all other pendulums in place in SCHLEGEISS dam, to better identify the special events of 1993.

It has also been shown that a simple deterministic model is not able to provide predictions with an accuracy comparable to that obtained with CONDORpc. To do so would require a number of data to be taken into account, with at least the full temperature field inside the dam Vs time, and time effects could be properly represented only with additional data on the material non-linear properties. The amount of work and data corresponding to such advanced model are out of scale with that mobilised by statistical approaches. Structural models should therefore **not** be used **alone** for interpreting monitoring readings. Their best use is at a next stage, when direct readings have already been interpreted with statistical analyses and summarised into strong trends. Only deterministic models are able to **explain** phenomena previously **identified**. They can be focused on specific aspects of the dam behaviour, and the engineer knows at this time which effects and which parameters are to be considered. For example, the non-linear response of the dam under exceptional hydrostatic loads such as that of 1993 could be more precisely investigated. This is the concept of the MODAP, the Model Accompanying a Project, set up at the time of the project (or later in the case of old dams) and progressively enhanced with features best suited to take into account the output of statistically interpreted monitoring, whose examples now exist for major dams (Carrere [1], Zenz [4]).

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Appendix

Presentation of the Hydrostatic-Seasonal-Time (HST) statistical method

The HST method was originally developed by Électricité de France (EDF). For more than thirty years it has been used to monitor the largest French dams and is now a standard. Coyne et Bellier and Ingema (Morocco) have developed the Condor II expert system for easy implementation of this method; it has a user-friendly interface for pre-processing and post-processing of monitoring data.

The HST method involves creating and periodically updating (typically every 2, 4, or 6 years) models of "hydrostatic, seasonal, and temporal" behaviour validated by statistical analysis, and using them for instant appreciation of new readings.

The readings accumulated from instrumenta installed on the dam are used to
i) appreciate reversible phenomena (H, linked to the reservoir level, and S, a seasonal term covering all the thermal effects factored to their mean annual variations, including the effect of seasonal rains where applicable),

ii) qualify the scatter of readings,
iii) detect and qualify any drift over time which could indicate significant evolution in the dam; this point is particularly important for periodical diagnoses.

Separation into one-variable functions makes it possible to independently analyse and, above all, qualify the effects of each parameter on the dam behaviour. In practical terms, the functions are determined through a multi-linear regression, where the raw measurement, MR, can be expressed as follows:

$$MR = f(Z) + g(S) + h(T) + R$$

where Z is the normalised reservoir level, S the season, T the time, and R the residual (unexplained divergence).

The corrected measurement, MC, is obtained by subtracting reversible effects from the raw measurement, which returns it to identical reservoir and seasonal conditions:

$$MC = MR - f(Z) - g(S) = h(T) + R$$

Scatter in raw and corrected measurements are characterised by the standard deviations (VaB and VaC) of these readings from their mean values. These standard deviations are involved in the definition of the global coefficient of correlation, CgC, and of the global coefficient of explanation, CgE, which represent the quality and validity of the model.

$$CgC = \sqrt{1 - \left(\frac{VaC}{VaB}\right)^2} \qquad CgE = 1 - \frac{VaC}{VaB}$$

Although the global correlation coefficient is often used by statisticians, the linearly-varying global coefficient of explanation says more to engineers; it characterises the gain in scatter due to the correction law, and corresponds to the part of variations in raw readings that can be explained by the model.

The width of the band of scatter of the raw readings and corrected readings can be expressed by 5 VaB and VaC standard deviations respectively. Ninety-nine percent of the measurements are within ±2.5 standard deviations.

A new reading MR can be immediately diagnosed by using correcting functions H and S and by comparing the corrected value MC with previous corrected measurements and with the corrected model graph (function T solely). If the residual for this measurement is within the ±1 residual standard-deviation band, it is deemed to be "normal"; if it is between ±1 and ±2.5 standard deviations, it is deemed to be "acceptable"; outside this range it is considered to be "abnormal".

The quick response of the diagnosis makes it possible to immediately take a new reading for any apparently abnormal measurement. If the anomaly is confirmed, it must be checked whether the deviation concerns a temporally and spatially isolated reading (damaged instrument or localised phenomenon), or a particular group of instruments (problem on a transmission line or, on the contrary, significant drift on one side of the dam or in a particular cross-section), or whether the deviation gets significantly larger over time.

In the latter case, the validity of the existing model must be questioned and a new test model is then generated, incorporating the input of the latest readings. At this time, the new model must be given a new interpretation, i.e. the values and the variations of its parameters (hydrostatic, seasonal and time effects) must be qualified again.

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A Simplified deterministic Model of Arch Dam Displacement based on the Technique of "Monothermometric Influence Coefficients"

ICOLD-TECHNICAL COMMITTEE ON COMPUTATIONAL PROBLEMS OF DAM ANALYSIS AND DESIGN

VI° BENCHMARK-WORKSHOP ON COMPUTATIONAL PROBLEMS OF DAM ANALYSIS AND DESIGN

SALZBURG, 17-19 OCTOBER 2001

THEME C – INTERPRETATION OF MEASUREMENTS FOR THE SCHLEGEIS ARCH DAM

Michele FANELLI, consultant
Gabriella GIUSEPPETTI, executive at ENEL.HYDRO

A SIMPLIFIED DETERMINISTIC MODEL OF ARCH DAM DISPLACEMENTS BASED ON THE TECHNIQUE OF "MONOTHERMOMETRIC INFLUENCE COEFFICIENTS"

SYNOPSIS

A rational modelling of the displacements caused by concrete temperature variations (to be added to those caused by variations in hydrostatic load) can be achieved using the theoretical framework introduced by FANELLI and GIUSEPPETTI (1976) under the denomination of 'monothermometric influence functions". In this approach a formal space/time separation of the FOURIER heat-conduction equation is effected under the assumption of periodic temperature variations. This allows to express the time dependence of the temperature at any point as a linear function of the measured temperatures (at a limited number of points inside the dam volume) and of their time-derivatives, the coefficients of this linear relationship being the space functions obtained from the formal space-time separation. As a consequence it is evident that, by introducing these same basic temperature space-distributions as the thermal input to a suitable structural analysis code, it becomes possible to compute, once and for all, influence coefficients (separately for the thermometric readings and for their time-derivatives) which allow to build up 'a priori' estimates of the thermal components of the structural displacements by using directly the measured temperatures (and their time-derivatives). The static components are evaluated –as usual- by building up, through repeated structural analysis, suitable analytic functions of the water level.
In the present paper the technique of 'monothermometric influence functions" is applied in a simplified context to hindcast the Schlegeis arch dam displacements, and it is shown that in spite of the simplifications introduced the model estimates of displacements can be made to follow closely –after calibration of the material constants- the corresponding measurements. After calibration of the constants using past data, the influence coefficients can be used to forecast in real time the expected displacements, with the aim of comparing them to the observed values and thus giving an objective yardstick to the monitoring activities.
The interest of the present study lies, therefore, in the demonstration that a satisfactory forecasting of expected displacements can be achieved not only with the powerful Finite Element or Boundary Element methods, but also by using much simpler software platforms, whose low cost and remarkable ease of use put rational displacement forecasting by deterministic models within reach even of small, limited-budget organisations and underprivileged countries.
The method may be expected to incur some loss of efficiency in the case of yearly cycles (both in water level and in temperature variations) markedly departing from periodicity; also, the necessity to obtain numerical estimates of the temperature time-derivatives calls for some caution, entailing the use of suitable numerical filters. However, in the Schlegeis case these delicate points did not appear to unduly affect the results.

1 – GENERAL CONSIDERATIONS, BASIC ASSUMPTIONS

Experienced dam engineers know very well that the setting up of a mathematical model intended to interpret in rational terms a physical phenomenon, starting from 'first principles', entails in any case a schematisation of all the complexities of the real world, and therefore leaves wide margins of choice among different options available to the modeller.
Considering this, and running somewhat contrary to the prevailing trends, the Authors asked themselves what kind of results could be achieved, were one to put oneself in the frame of mind to choose, at every stage of the model formulation, the simplest possible assumptions, identified by isolating the predominant factors at play in the physical phenomena involved and remorselessly discarding all the more sophisticated options.
Thus, confronted with Theme C of the present Benchmark-Workshop, they decided not only to try and use, as a structural model, the DESARC 3.2® code (which is based on a RITTER-like formulation, the foundation compliance being modelled through the VOGT coefficients; the underlying model allows for merely a couple of dozens of degrees of freedom at most), but even to model the temperature space/time variations in the concrete mass as being produced, at each elevation, by a *locally one-dimensional* heat flow along the upstream/downstream horizontal direction; this heat flow, furthermore, being assumed to be *periodic* (with a yearly period). However, within the framework of these rather crude shortcuts, the Authors wished to preserve as rational an approach as possible, e. g. by respecting, in the modelling of the thermal components of displacements, the FOURIER one-dimensional heat-conduction equation; and by considering that in order to correctly synthesise the space/time temperature distribution from the observed thermometric readings not only the temperatures themselves, but also their time-derivatives should be taken into account (see FANELLI and GIUSEPPETTI, 1976).
These drastic simplifications in basic assumptions adopted by the Authors were not motivated by sheer curiosity: in fact, were one to demonstrate that such stripped-down models could still interpret with reasonable accuracy the observed displacements, the interesting by-product of such a study would be the possibility to use very fast, economical software platforms for on-line monitoring and safety control. This could present an interest for small dam operators as well as for those charged with dam monitoring in underprivileged countries.
Besides, the Authors think that the simplified mathematical models are interesting in themselves, allowing the engineer a firmer, more intuitive grasp over the workings of the mathematical machinery; something that the heavy number-crunching mainstream codes do not easily allow.
The development of this exercise was quite exciting, and the results left the Authors with the preliminary impression that such apparently crude models might, indeed, be used quite efficaciously for displacement forecasting; however, the observed displacements for the year 2000 having not been provided by the B.-W. organisers (and quite rightly so!), the final proof of the pudding, as always, will lie in the eating... (see paragraph **3.6**, Figures 1, 2; paragraph **4**, Figures 3, 4).

2 – THE CONCEPT OF 'MONOTHERMOMETRIC INFLUENCE FUNCTIONS'

In a concrete dam where the temperatures are monitored by embedded thermometers the first problem confronting the analyst trying to set up a deterministic behaviour model consists in reconstructing the continuous thermal field from the discontinuous information provided by the instruments. This problem can be tackled in different ways: for instance, simple space-interpolation functions could be applied to the instantaneous measurements of temperature. However, it is readily seen that this approach cannot, in general, satisfy the FOURIER heat-conduction equations, except in the trivial case of stationary conditions, which is clearly without interest for dams; in fact, only the temperature variations with respect to the average stationary state are implicitly considered in the following.
FANELLI and GIUSEPPETTI (1976) showed that it is possible to exactly reconstruct the thermal field from the thermometric measurements $\vartheta_j(t)$, under the assumption of periodic temperature variations, from the equation:

$$\vartheta(x,y,z,t) = \sum_j f_j(x,y,z).\vartheta_j(t) + \sum_j \varphi_j(x,y,z).\frac{d\vartheta_j}{dt} \quad , \qquad (2.1)$$

where the space functions $f_j(x,y,z)$, $\varphi_j(x,y,z)$ satisfy conditions (2.2), derived by imposing that (2.1) should satisfy the FOURIER heat-conduction equation $a.\nabla^2\vartheta = \frac{\partial\vartheta}{\partial t}$:

$$\left.\begin{aligned} a.\nabla^2 f_j &= -\omega^2.\varphi_j \\ a.\nabla^2\varphi_j &= f_j \end{aligned}\right\} \qquad (2.2)$$

a being the thermal diffusivity of concrete (in the order of $0.004 m^2/h$ [1]) and ω the circular frequency of the temperature variations: $\omega = \frac{2.\pi}{T}$, if T is the corresponding period (usually one year, or 8760 hours)[2]; the boundary conditions for the aforementioned space functions are:

$$\left.\begin{aligned} f_j(P_k) &= \delta_{jk} \\ \varphi_j(P_k) &\equiv 0 \end{aligned}\right\} \qquad (2.3)$$

at the thermometric locations P_k , δ_{jk} being the KRONECKER delta.

The problem is then reduced on one hand to defining the space functions $f_j(x,y,z)$, $\varphi_j(x,y,z)$, on the other hand to finding effective means to estimate the time-derivatives $\frac{d\vartheta_j(t)}{dt}$ from the measurement data. Besides, the basic assumption of periodic thermal conditions should be critically examined, trying to ascertain the errors that can be produced in the results as a consequence of the possible departures of real conditions from the strict periodicity.

3 – THE BUILDING UP OF THE 'MONOTHERMOMETRIC INFLUENCE COEFFICIENTS'

As a consequence of (2.1), it is evident that by introducing the unit space functions $f_j(x,y,z)$, $\varphi_j(x,y,z)$ (pertaining to the generic thermometer T_j located at P_j) as distinct thermal inputs in a structural analysis code, the displacements thereby obtained at the points of interest for monitoring,

$$\left.\begin{aligned} &CM_{f,j} \\ &CM_{\varphi,j} \end{aligned}\right\} \qquad (3.1)$$

are the sought-after 'monothermometric influence coefficients', such that the thermal component of the corresponding displacement is given in our model by:

$$\delta(t) = \sum_j CM_{f,j}.\vartheta_j(t) + \sum_j CM_{\varphi,j}.\frac{d\vartheta_j(t)}{dt} \qquad (3.2)$$

Therefore the knowledge of (3.1) solves –consistently with our starting assumptions- the problem of using directly the measured temperatures[3] to forecast, by means of pre-computed influence coefficients, the thermal part of the dam behaviour; the only difficulty being the necessity to build up, from the available measurements, also reliable, bias-free estimates of the corresponding time-derivatives.

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[1] That the value of a is not far from the quoted value in the case of the Schlegeis arch dam is confirmed from comparison of the observed variations at H 12 MI (the variations at H 15 MI are to small to be of any use) with those deduced from the variations at H 12 UP and H 12 DO [by eq. (2.1)] under the assumption $a = 0.004 m^2 h^{-1}$.

[2] It can be shown that the monothermometric influence coefficients here defined show relatively small variations with the frequency of thermal variations, provided that the thickness is greater than about 10-15 m; this explains why the technique works also for thermal variations that include harmonics of frequency higher than the annual one.

[3] The above developed treatment would apply, as stated, only to the zero-average, periodic temperature variations; by introducing instead into (3.2) the raw thermometric readings, as ease of use suggests, a constant offset is generated, which is taken care of by the calibration of the model constants, see further on.

3.1 – THE SPACE/TIME SEPARATION IN THE FOURIER ONE-DIMENSIONAL EQUATION: THE BASIC UNIT TEMPERATURE DISTRIBUTIONS (EACH RELATED TO THE READINGS OF ONE SINGLE THERMOMETER)

A favourable circumstance occurring in arch dams, as a consequence of the slenderness of their crown section, is the fact that the three-dimensional heat conduction across the thickness can be approximated with reasonable accuracy, at each elevation, by a one-dimensional heat flow along the local horizontal direction, x . Thus, in place of (2.2), we can write for each thermometric section the approximate relationships:

$$\left.\begin{aligned} a.\frac{d^2 f_j}{dx^2} &= -\omega^2.\varphi_j \\ a.\frac{d^2 \varphi_j}{dx^2} &= f_j \end{aligned}\right\} \quad , \qquad (3.1.1)$$

with boundary conditions like eq. (2.3), which can be solved analytically when there are but two thermometers in each horizontal thermometric section (one near the upstream face and one near the downstream face).

The general solution of (3.1.1) can be found without difficulty:

$$f_j(x) = A_j.Ch(\alpha.x).\cos(\alpha.x) + B_j.Ch(\alpha.x).\sin(\alpha.x) + C_j.Sh(\alpha.x).\cos(\alpha.x) + D_j.Sh(\alpha.x).\sin(\alpha.x)$$
$$\omega.\varphi_j(x) = -D_j.Ch(\alpha.x).\cos(\alpha.x) + C_j.Ch(\alpha.x).\sin(\alpha.x) - B_j.Sh(\alpha.x).\cos(\alpha.x) + A_j.Sh(\alpha.x).\sin(\alpha.x)$$
(3.1.2)

where $$\alpha = \sqrt{\frac{\omega}{2.a}} \qquad (3.1.3)$$

($\alpha \cong 0.3$ for the usual values of a , ω).

The four constants A_j, B_j, C_j, D_j appearing in (3.1.2) can be determined by imposing the boundary conditions:

$$\left.\begin{aligned} f_j(x_j) &= 1 \\ f_j(x_k) &= 0 \\ \varphi_j(x_j) &= 0 \\ \varphi_j(x_k) &= 0 \end{aligned}\right\} \qquad (3.1.4)$$

where x_j , x_k are the local abscissae of the two thermometers of the section (being e. g. $x = 0$ on the upstream face and $x = L(z)$ on the downstream face).

Given the available thermometric sections (there should be at least two of them, at different elevations: we assume these thermometric sections to be located, as usual, in the vertical crown section), it remains to synthesise plausible space distributions *all over the vertical crown section*: $F_j(x,z)$, respectively $\Phi_j(x,z)$, from each of the *local* x-wise unit distributions $f_j(x)$, $\varphi_j(x)$; this can be effected thanks to suitable shape functions in the variable z or $\zeta = \frac{z}{H}$ (H being the dam height), as will be explained in the next paragraph. The variations of the temperature distribution along the third coordinate, i. e. along the horizontal planes away from the vertical crown section, are disregarded in the absence of more detailed information, this being usually the case. In this way each of the unit temperature distributions gives rise to a space-wise completely defined thermal state, which can be used as already explained to generate the 'monothermometric influence coefficients' for the thermal component of the displacement of any given point.

3.2 – THE INTERPOLATION OF THE BASIC x-WISE DISTRIBUTIONS WITH RESPECT TO ELEVATION

Let us introduce non-dimensional coordinates ξ , ζ :

$\xi = \dfrac{x}{L(z)}$; $\xi = 0$ along the upstream face, $\xi = 1$ along the downstream face;

$\zeta = \dfrac{z}{H}$; $\zeta = 0$ at dam crest, $\zeta = 1$ at the dam-foundation interface on the crown vertical section.

Let us denote the discrete values taken up by ζ at the thermometric sections by ζ_m [4]; then let us introduce shape functions

$N_m(\zeta)$, as many as there are thermometric sections, satisfying the usual conditions for shape functions:

$$N_m(\zeta_n) = \delta_{mn} \quad \text{(KRONECKER delta)}$$

$$\sum_m N_m(\zeta) \equiv 1 \quad \text{for any } \zeta \text{ , } 0 \le \zeta \le 1 \qquad (3.2.1)$$

We assume next that the thermal state in the crown section, to be extended along horizontal planes to the whole dam, can be approximated as follows:

$$\vartheta(\xi,\zeta,t) = \sum_j N_m(\zeta).f_j\left(\xi,\zeta_{m(j)}\right).\vartheta_j(t) + \sum_j N_m(\zeta).\varphi_j\left(\xi,\zeta_{m(j)}\right).\frac{d\vartheta_j}{dt} \qquad (3.2.2)$$

Introducing the $2.m_{\max}$ 'unit' thermal inputs:

$$\left.\begin{array}{l} N_m(\zeta).f_j\left(\xi,\zeta_{m(j)}\right) \\ N_m(\zeta).\varphi_j\left(\xi,\zeta_{m(j)}\right) \end{array}\right\} \qquad (3.2.3)$$

into any suitable structural analysis code, the 'monothermometric influence coefficients'

$$\left.\begin{array}{l} CM_{f,j} \\ CM_{\varphi,j} \end{array}\right\} \quad , \qquad (3.2.4)$$

valid under our simplifying assumptions and to be used in the computation of the thermal component of the displacements:

$\delta(t) = \sum_j CM_{f,j}.\vartheta_j(t) + \sum_j CM_{\varphi,j}.\dfrac{d\vartheta_j(t)}{dt}$, see (3.2), are at last obtained.

It is to be noted that the above outlined procedure is straightforward and does not require to follow in time the numerical solution of the heat-propagation equations.

3.3 – THE COMPUTATION OF THE MONOTHERMOMETRIC INFLUENCE COEFFICIENTS FOR THE SCHLEGEIS ARCH DAM

In our exercise pertaining to the case of the Schlegeis arch dam, the above outlined simplified procedure was followed, choosing as thermometers the two couples H 12 UP, H 12 DO at elevation 1750.65 ($\zeta = 0.247$) and H15 UP, H 15 DO at elevation 1677.15 ($\zeta = 0.808$). The middle thermometers at each of these two sections, H 12 MI and H 15 MI, need not be considered (the time variations of the corresponding temperatures are, anyway, quite negligible in comparison with those of the other thermometers; see footnote 1).

Parabolic shape functions N_m satisfying the condition $\dfrac{dN_m}{d\zeta} = 0$ at crest ($\zeta = 0$) were chosen.

The basic unit thermal inputs thus defined, see paragraph 3.2, have been computed and applied in the framework of the DESARC 3.2® software package, obtaining the following monothermometric coefficients for the radial component of the crest displacement (positive in the downstream direction):

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[4] An index m different from j , the index used for the thermometers, is here introduced because at each thermometric section (hence at a single value of ζ) there are two thermometers. Obviously $m = m(j)$.

TABLE 1 – MONOTHERMOMETRIC COEFFICIENTS FOR THE SCHLEGEIS ARCH DAM (UNCALIBRATED VALUES)

	H12 UP, CM_f	H12 UP, $\omega.CM_\varphi$	H12 DO, CM_f	H12 DO, $\omega.CM_\varphi$	H15 UP, CM_f	H15 UP, $\omega.CM_\varphi$	H15 DO, CM_f	H15 DO, $\omega.CM_\varphi$
mm/°C	-0.5324		-1.4421		-0.1267		-0.0330	
mm/°Crad		0.3349		0.3776		0.0371		0.0186

(In the above table the products $\omega.CM_\varphi$ have been listed, in place of CM_φ, to avoid having to deal with very large numerical values; these values can be used as such provided the temperature derivatives are effected with respect to 'non-dimensional time' $\omega.t$, one year corresponding thus to $2.\pi$ radians).
The listed values were obtained taking for the coefficient of thermal dilatation the value of $8.10^{-6}(°C)^{-1}$ as suggested in the enunciation of Theme C of the 6th Benchmark-Workshop. [Successive calibration of the model constants (see following paragraph) might suggest that a different value of the thermal dilatation coefficient should be used].

3.4 – THE CALIBRATION OF THE MODEL CONSTANTS

The comparison between observed displacements and model forecasts have to be carried out, by necessity, on the *total* displacements, which are the sum of the component tied to the variations in water level and those caused by the temperature variations. Besides, a constant offset of the forecast displacements is to be expected, see footnote 2; a drift term (irreversible component) may also be present. Consequently, the *total* displacement forecast by the mathematical model can be written down as:

$\delta_{TOT} = P + D + R.F(q) + S.\delta(\vartheta)$, where $D(t)$ is the drift term, (3.4.1)

$P,\ R,\ S$ = calibration constants , (3.4.2)

$F(q)$ = a suitable function of the water level $q(t)$, (3.4.3)

obtained by repeated application of the hydrostatic load with varying impoundment elevations, and

$$\delta(\vartheta) = \sum_j CM_{f,j}.\vartheta_j(t) + \sum_j CM_{\varphi,j}.\frac{d\vartheta_j(t)}{dt}$$ as already discussed at length. (3.4.4)

Comparison of the forecast values, δ_{TOT} , with the observed displacements δ_{OBS} over a suitable stretch of time (preferably several years) will generate a time-series of differences $V(t)$:

$V(t) = \delta_{OBS}(t) - \delta_{TOT}(t)$, (3.4.5)

which are functions of the values assigned to the calibration constants $P,\ R,\ S.$ and of the model adopted for $D(t)$.

By imposing the usual least-squares condition:

$$\sum_t [V(t)]^2 = \min$$, (3.4.6)

namely
$$\begin{cases} \dfrac{\partial \sum_t [V(t)]^2}{\partial P} = 0, \\ \dfrac{\partial \sum_t [V(t)]^2}{\partial R} = 0, \\ \dfrac{\partial \sum_t [V(t)]^2}{\partial S} = 0 \end{cases}$$, (3.4.7)

the 'best fit' values for the model constants are determined.

3.5 – THE PROBLEM OF THE EVALUATION OF THE TEMPERATURES' RATE OF CHANGE

As stated earlier, the present approach is valid, strictly speaking, for periodic variations of temperature – a circumstance that cannot be expected to hold exactly in actual practice. The necessity to evaluate rates of change of the thermometric readings in order to apply the present methodology highlights this contradiction between theory and reality [5].

Therefore two questions arise: on one hand to obtain some estimate of the errors that can affect the present model results as a consequence of the irregularities of the thermal cycle, on the other hand how to evaluate the time-derivatives in (3.4.4) so as to minimise those same errors. Past practice suggests that in order to reach the latter goal a compromise should be struck between a strictly 'local' evaluation of the time-derivatives in question and a more 'smoothed' one .

Several different numerical devices can be imagined to produce an estimate of the needed time-derivatives having the required properties; some of them will be briefly touched upon in the following.

a) Harmonic analysis – This may seem the 'best' means to obtain the amplitude and phase of the yearly 'fundamental' thermal wave (of course separately for each of the thermometric time series) and from that obtain the required rate of change 'as if' the signal were reduced to the fundamental in question. However, to get a reliable estimate a rather long stretch of each time series should be taken into account, which besides posing a problem for 'real time' monitoring is also bound to impair the local representativity of the derivative estimate.

b) Piecewise, local fitting of amplitude and phase – By dividing each available time series $\vartheta(t)$ into successive intervals of the order of one year or more, and defining for each interval from t_ℓ to $t_{\ell+1}$ shape functions such as, e. g.:

$$\left.\begin{aligned} M_\ell(t) &= \frac{t_{\ell+1}-t}{t_{\ell+1}-t_\ell} \\ M_{\ell+1}(t) &= \frac{t-t_\ell}{t_{\ell+1}-t_\ell} \end{aligned}\right\} \quad \text{for} \quad t_\ell \le t \le t_{\ell+1} \quad , \qquad (3.5.1)$$

a least-squares best-fit between the time series and the function:

$$\sum_\ell M_\ell(t).[\Theta_{s\ell}.\sin(\omega.t)+\Theta_{c\ell}.\cos(\omega.t)] \qquad (3.5.2)$$

allows to determine at each instant t_ℓ the unknowns $\Theta_{s\ell}$, $\Theta_{c\ell}$ and thereafter to evaluate locally 'best-fitting' amplitude and phase of a yearly cycle:

$$\left.\begin{aligned} \Theta_\ell &= \sqrt{\Theta_{s\ell}^2+\Theta_{c\ell}^2} \\ \psi_\ell &= arctg\frac{\Theta_{c\ell}}{\Theta_{s\ell}} \end{aligned}\right\} \quad , \qquad (3.5.3)$$

such that for $t_\ell \le t \le t_{\ell+1}$:

$$\left.\begin{aligned} \vartheta(t) &\cong \Theta_\ell.\sin(\omega.t+\psi_\ell), \\ \frac{d\vartheta}{d(\omega.t)} &\cong \Theta_\ell.\cos(\omega.t+\psi_\ell) \end{aligned}\right\} \quad . \qquad (3.5.4)$$

c) Empirical 'almost local' filters – Let us remark that since

[5] It may seem, indeed, contradictory to derive influence coefficients based on the assumption of periodicity and to use 'local' time derivatives for time series which are not strictly periodic; in practice it has been shown that results good enough for monitoring purposes can be obtained with a carefully balanced 'mixed' approach in which the time series is somewhat smoothed so as to attenuate the local character of the derivative estimates.

$$\frac{d\vartheta}{d(\omega.t)} = \frac{1}{\omega}.\frac{d\vartheta}{dt} \cong \frac{T}{2.\pi}.\frac{\Delta\vartheta}{\Delta t} \quad , \qquad (3.5.5)$$

taking $\Delta t = \frac{T}{2.\pi}$ it comes $\frac{d\vartheta}{d(\omega.t)} \cong \Delta\vartheta\left(\Delta t = \frac{T}{2.\pi}\right)$, (3.5.6)

and noting that $\frac{T}{2.\pi} = \frac{365}{2.\pi} \cong 58$ days the apparent conclusion is that one could use, instead of the derivative $\frac{d\vartheta}{d(\omega.t)}$, the finite increment $\Delta\vartheta$ over 58 days (the value 29 days after the date of interest minus the value 29 days before the same date). Actually, the results produced by such a procedure could suffer from irregularities occurring around the two extreme dates of the 58 days interval; therefore, it is better to use a numerical filter based on a suitable convolution integral over the 58 days period, e. g.

$$\Delta\vartheta\left(\frac{T}{2.\pi}\right)\Big|_t \cong 0.110118.\int_{t-29}^{t+29}\vartheta(\tau).\sin\left(\frac{\pi.\tau}{29}\right).dt \qquad (t \text{ in days}), \qquad (3.5.7)$$

which yields exact results if $\vartheta(t)$ is a sinusoid with a yearly period, or a similar formula satisfying three essential requisites: *i*) that a constant base value should give null result (the convolution factor should therefore be an odd function of the time difference from the date of interest), *ii*) that the random irregularities over the integration period should tend to cancel out. and *iii*) that the influence over the result should be maximum for intermediate values of the date difference with respect to the central instant, decreasing till zero for values too near or too far from this date. This is the procedure that the Authors followed in the Schlegeis arch dam application illustrated in paragraph **4**.

d) Empirical adjustments of the finite increments – Taking as at the beginning of c) the finite increments

$$\Delta\vartheta\left(\frac{T}{2.\pi}\right) = \vartheta(t+29) - \vartheta(t-29) \qquad (3.5.8)$$

for all the dates t over a whole period (1 year), a two-step manual adjustment is made on all the values in order to give them a regular periodic behaviour that does not affect their positive/negative average over the two half-periods.

It is to be remarked that procedures a), c), d) pose problems for real-time forecasting, insofar as all of them require to know values posterior in time to the date of interest; whereas procedure b) can be applied to yield results as soon as a new value is obtained, albeit wit lesser accuracy than for past intervals dating back from more than a half-period.

3.6 – SOME PRELIMINARY RESULTS FOR THE SCHLEGEIS ARCH DAM

A first spot application was carried out for a chosen one-year period, using approach d) of previous paragraph and assuming that no appreciable drift was present: $D(t) \cong 0$.

Fig. 1 shows a comparison between forecast and observed displacements at the mid-date of each month for the year 1998, after calibration of the three constants P, R, S . The values obtained by the calibration procedure and the impoundment function were the following:

$$\left.\begin{array}{l} P = 35.7mm \\ R = 0.626 \\ S = 1.149 \end{array}\right\} \quad , \quad \left.\begin{array}{l} F(q) = Exp\left(2.62.\sqrt{w+0.146} - 3.283\right) \\ w = \dfrac{q-1652}{13.1} - 1 \end{array}\right\} \quad , \quad q > 1663.2$$

(3.6.1) , (3.6.2)

The above impoundment function $F(q)$ was obtained from a least-square adjustment with respect to the values obtained from computations effected for discrete water elevation values q.

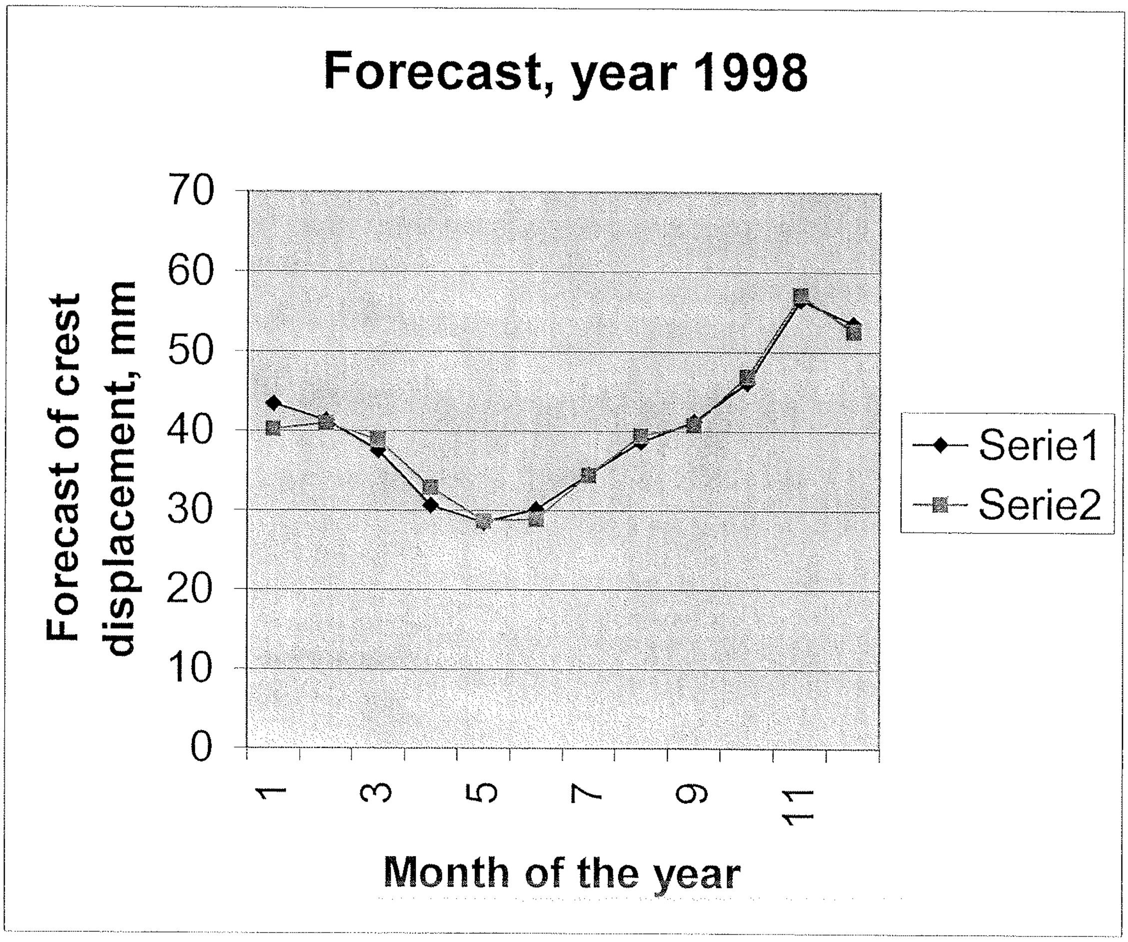

FIG. 1 – YEAR 1998
COMPARISON BETWEEN MEASUREMENTS (Series 1) AND MODEL FORECAST (Series 2)

The values obtained by calibration seem to indicate that the effective value of the Young modulus is higher than the indicated value of 250,000 $kgf.cm^{-2}$, and that the effective coefficient of thermal dilatation is also higher than the indicated value of $8.10^{-6}(°C)^{-1}$.
With these same values of the calibration constants a forecast was attempted for the year 2000 (see following TABLE 2):

TABLE 2 – DETAILS OF DISPLACEMENT FORECAST FOR MID-MONTH DATES, YEAR 2000

month	Water level	H12UP °C	DH12UP °C/rad	H12DO °C	DH12DO °C/rad	H15UP °C	DH15UP °C/rad	H15DO °C	DH15DO °C/rad	**Forec. displ. mm**
1/00	1746.77	4.1	-0.9	-1.7	-2.975	4.8	-1.7	-1.3	-2.575	**51.2**
2/00	1733.59	3.8	0.9	-1.0	0	4.3	-1.55	-1.7	0	**45.0**
3/00	1711.38	4.4	1.825	0.1	2.975	4.0	-1	-0.5	2.575	**37.8**
4/00	1697.27	5.6	3.175	2.0	4.575	3.8	0	0.8	4.2	**33.3**
5/00	1721.54	9.0	3.175	6.2	4.75	4.0	1	4.6	4.65	**28.5**
6/00	1737.25	10.1	1.825	8.2	4.575	4.9	1.55	6.6	4.2	**29.8**
7/00	1746.96	10.5	0.9	8.4	2.975	5.9	1.7	7.6	2.575	**33.7**
8/00	1765.09	9.2	-0.9	9.0	0	6.7	1.55	8.1	0	**48.7**
9/00	1762.72	8.5	-1.825	8.3	-2.975	7.3	1	7.4	-2.575	**45.7**
10/00	1768.82	8.1	-3.175	6.7	-4.575	7.5	0	6.1	-4.2	**55.1**
11/00	1764.80	7.0	-3.175	4.1	-4.75	6.9	-1	3.7	-4.65	**54.8**
12/00	1742.97	6.0	-1.825	3.4	-4.575	5.9	-1.55	2.6	-4.2	**37.7**

CMf		**-0.5324**		**-1.4421**		**-0.1267**		**-0.0330**		
CMphi			**0.3349**		**0.3776**		**0.0371**		**0.0186**	

These forecasts are graphically depicted in the following diagram (Fig. 2)

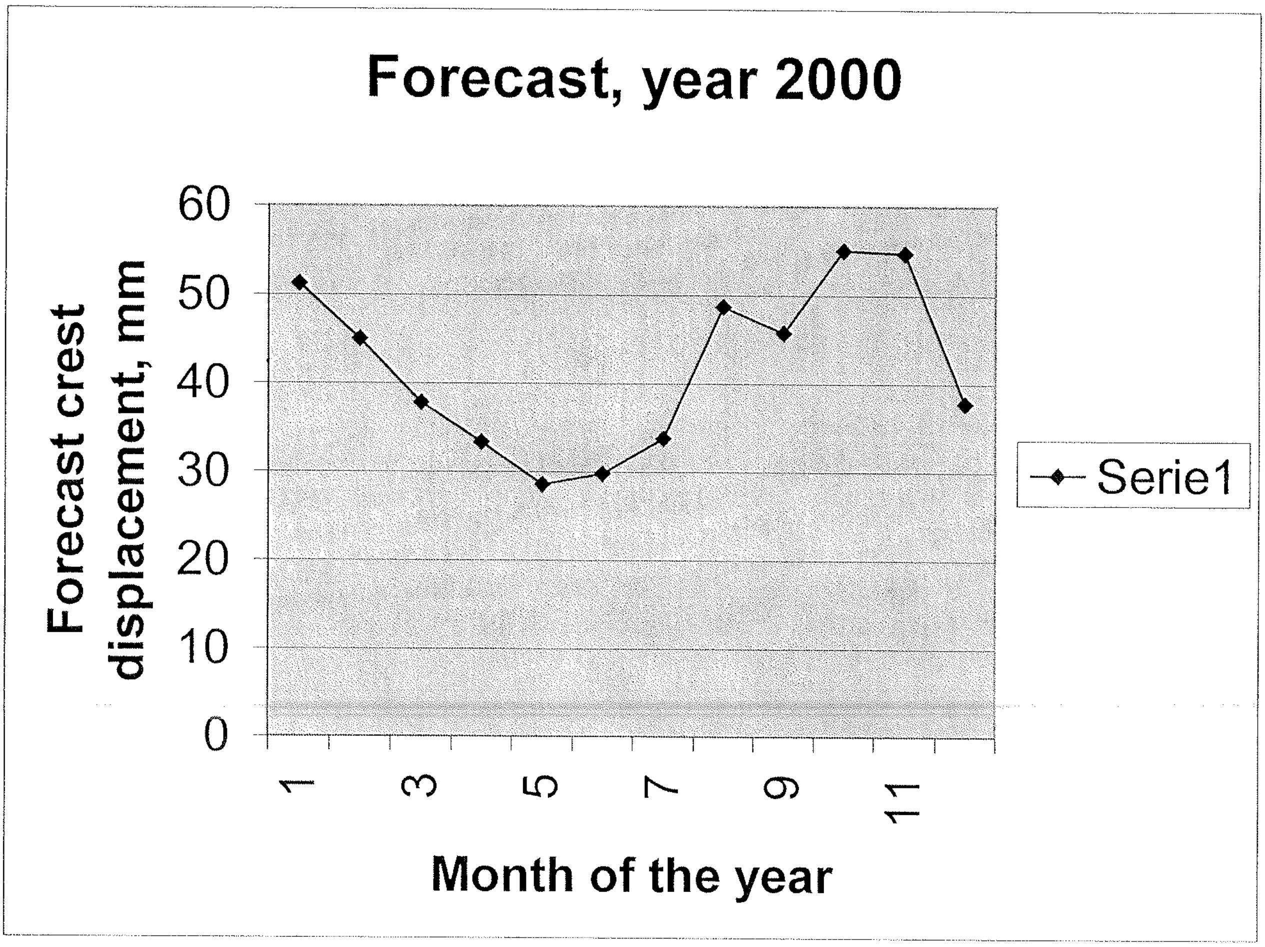

FIG.2 – YEAR 2000
MODEL FORECAST (Series 1) FOR MID-MONTH DATES

Comparisons between measurements and model forecasts were also effected for years 1992 and 1995, in which cases the least-squares values of the calibration constants were slightly different from those found for year 1998; this would seem to indicate –assuming that the mathematical model is reliable- either a drift phenomenon or some discontinuity in the dam behaviour.

Successively, an analysis for the entire period of observation 1992-1998 was carried out using a more refined model, in order to respond more accurately to the questions posed in the framework of the present B.-W. This more complete application (which includes also a drift model) and the relevant model will now be described.

4 – THE MODEL ADOPTED FOR THE COMPLETE ANALYSIS OF THE SCHLEGEIS DAM DATA

This model (which was defined by successive refinements tending to achieve smaller and smaller deviations between displacements forecasts and measurements over the 1992-1999 calibration period) was based on an identification of the amount of the linear and the parabolic components of the elevation-wise temperature distribution[6] (the monothermometric coefficients having been computed both under the linear and the parabolic assumption). Moreover, it was assumed that these relative amounts could be periodically variable during the year and that it was better, for physical reasons, to keep these proportions distinct for the upstream and the downstream thermometers; this refined formulation called for a total of 7 calibration constants for the thermal component of the displacement. A drift term was also explicitly included in the model, based on the cubic splines described in FANELLI, GIUSEPPETTI and MAZZA', 2000; the period 1992 to 1999 was divided into ten equal intervals for the purpose of drift identification . Taking into account the 13 constants required to define the amplitudes of those splines and the single calibration constant of the impoundment component, the total number of calibration constants for this second-generation model amounted to 21. [It might be contended that, this number being quite large, the proposed model is in practice not much different from a statistical one; however, it should be considered that the structure of the model is derived from a rational, deterministic formulation and thus contains a deeper insight than a truly statistical approach].
In Figures 3 and 4 the main results of this analysis are presented.

[6] There being only two thermometric sections, these relative amounts could not be identified on the sole basis of the temperature measurements. The guess adopted for the preliminary trial illustrated in paragraph **3.6**, i. e. a parabolic law with zero derivative at crest, could not be assumed without further proof and indeed produced greater deviations.
[This rather cumbersome formulation could be avoided altogether had the two thermometric sections been placed at 'optimal' elevations (approximately 1779.65 and 1719.65 for the Schlegeis dam), see FANELLI 2001].

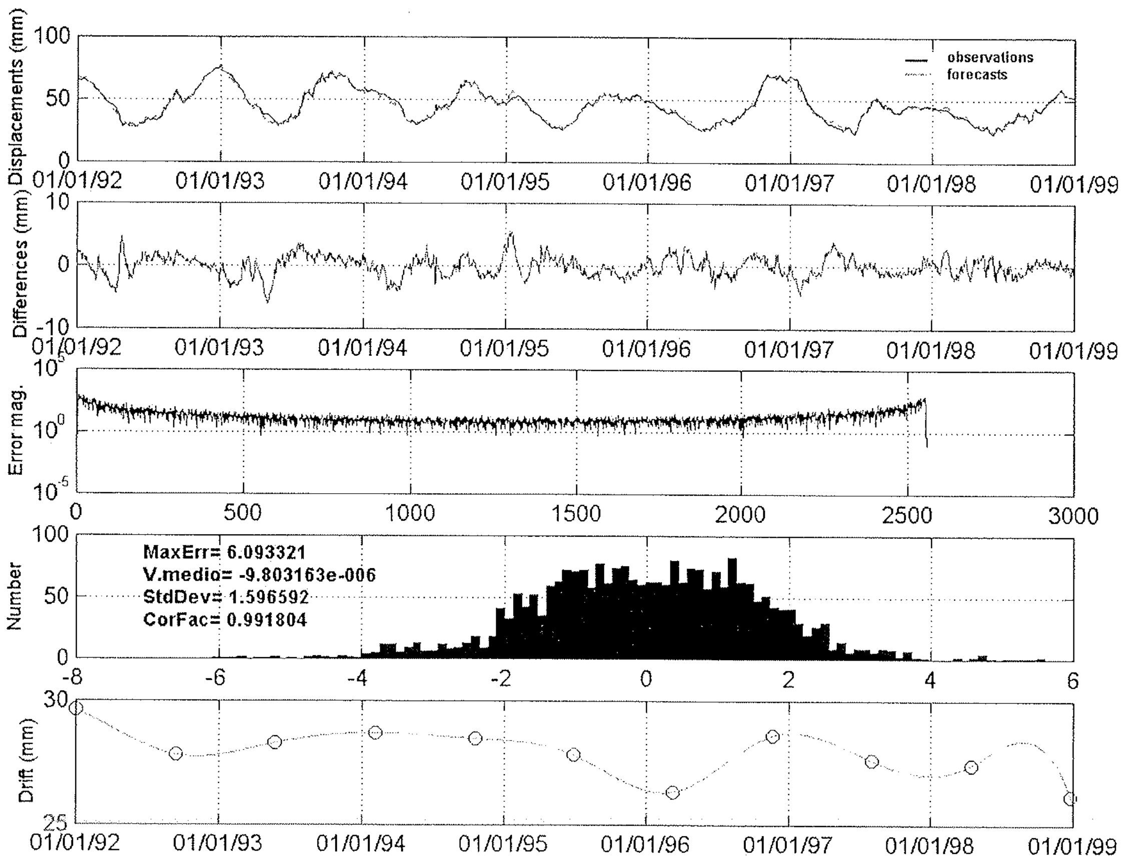

FIG. 3

From top to bottom:

-diagram of forecasts and observations, years from 1992 to 1998

-diagram of differences between forecasts and observations

-Fourier transform of the differences

-Frequency distribution of the differences

-Time-drift as identified by the model

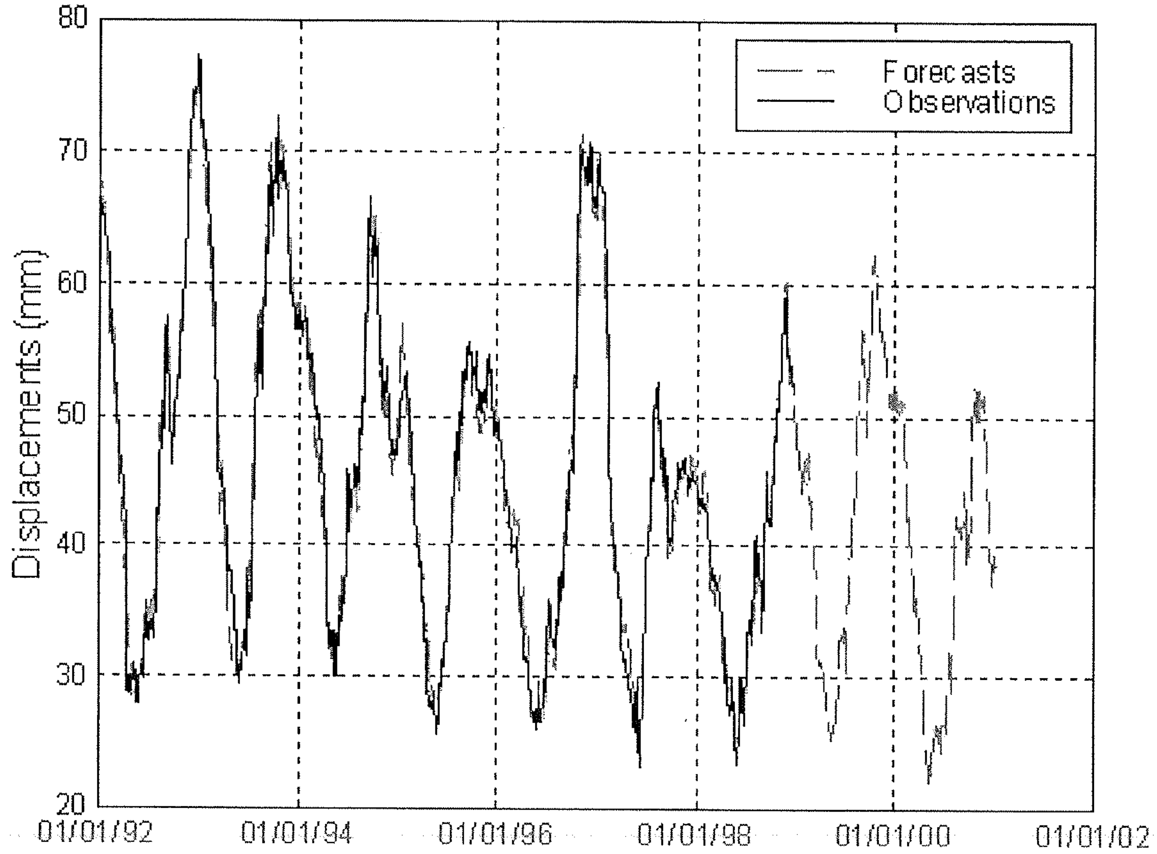

FIG. 4
Forecasts of the model, years from 1992 to 2000; for the period 1992-1998 also the measurements are plotted

The above results call for some comments.

FIG. 3 – The correlation index between forecasts and observations (0.992) is satisfactory; the differences between the model forecasts and the actual measurements appear to be sufficiently small (standard deviation 1.597 *mm*); their Fourier transform is sufficiently flat, as it is desirable insofar as the deviations should approximate a kind of 'white noise'; their frequency distribution, albeit satisfactory, departs somewhat from a Gaussian one, tending to be flatter in the centre and more abruptly terminated in the extreme tails (apart from some outliers). This fact, together with the visual inspection of the chronological diagram of the differences, might suggest that the first two years (1992 and 1993) are not homogeneous with the following ones[7]. In this connection some attempts were made to improve further the model, excluding from the calibration period the first two years (calibration effected over the period from 1994 to 1998); the results improved indeed, but only marginally (apart from the frequency distribution of the differences, which became more markedly 'normal-like'), so that it was deemed not worthwhile to adopt this modified model as the 'final' one, nor to illustrate in detail these results in the framework of the present paper.
The chronological behaviour of the identified drift is somewhat puzzling, presenting 'ups and downs' that do not lend themselves to a clear physical interpretation; however, this behaviour appeared to be remarkably stable in all the variants of the present model that were tried, thus lending credibility to the result in question. The physical causes of such a behaviour remain, however. to be investigated.

[7] Which is plausible since during the first years after construction the thermal transient caused by the dissipation of the cement hydration heat induces a thermal regime departing from the periodic one on which the present model is based.

FIG. 4 – The model forecasts appear, at a visual inspection, to follow closely and without appreciable time-lag the day-to-day chronological variations of the measurements over the calibration period, thus proving on one hand the efficacy of the 'monothermometric' influence coefficients, on the other hand the importance of including the effect of the time-derivative of the measured temperatures. Maximum deviation is about 6.1 *mm*, but almost always the deviations are much smaller than that (their standard deviation, as already mentioned, is about 1.6 *mm*, i. e. about 3% of the amplitude of displacement variations). It remains of course to be seen how well the model succeeds in following the 1999 and 2000 observations.

5 – SUMMING UP

In the present approach three favourable circumstances were taken advantage of in order to drastically simplify the complexities of the thermal behaviour:

- the displacements, being integral quantities, are scarcely sensitive to local details of the heat flow variations, so that even crude schematisations of the latter will give quite accurate results provided they capture the predominant features of the physical phenomenon;
- the slenderness of the crown section in arch dams suggests that, apart from appreciable perturbations near the water-air-concrete interface, the heat flow across the thickness is 'almost everywhere' predominantly one-dimensional along the upstream/downstream horizontal direction;
- the variations in air, water and concrete temperatures show an appreciable degree of periodicity from one year to the next one. The water level variations are of a less regular nature.

Advantage was taken of these circumstances to effect a straightforward space-time separation of the temperature variations and thus derive -with the help of adequate interpolation functions effecting the passage from the discrete information provided by the installed thermometers to a continuous distribution- coefficients of influence that could be computed once and for all for the thermal components of displacements. The particular formulation adopted allowed to build up influence coefficients each of which pertains to the temperature variations of a single control thermometer (both the instantaneous values and the relevant instantaneous rates of change need be considered), whence the denomination 'monothermometric influence coefficients'.

The whole procedure, furthermore, lends itself quite naturally to implementation in the framework of simplified structural analysis codes for arch dams, such as the DESARC 3.2® software package (although the latter was developed mainly for preliminary design and shape optimisation rather than for monitoring activities).

The application of these guidelines to the Schlegeis arch dam produced satisfactory results (after calibration of the model constants) for the years from 1992 to 1998, in which both external action data and displacement data were made available; some of the results obtained in a preliminary trial (with only three calibration constants) were shown in the preceding paragraph **3.6** , while a more complete analysis of the entire period from 1992 to 1998 (with a drift term included and 21 calibration constants), as well as the relevant projections for the years 1999 and 2000, were illustrated in paragraph **4.**

While an eventual judgement of the practical value of the proposed approach will have to wait until after the comparison of these last projections with observational displacement data (undisclosed prior to the B.-W.) will have been effected, the simplicity and ease of implementation of the procedure warrant, in the Authors' opinion, further study and critical appraisal. Meanwhile, a few practical suggestions can be drawn from the present exercise:

- at least two thermometric sections (but probably not more than three are needed) should be used for deterministic modelling of thermal displacements, the uppermost section being near the crest and the lowermost one near the dam/foundation interface; in each section two thermometers are sufficient, one near to the upstream face and the other near the downstream face; however, the collocation of these thermometers both as regards elevation and distance from the dam face should be carefully chosen in order to optimise the model accuracy (see FANELLI 2001)[8]:

- the thermal diffusivity of the concrete should be known; alternatively, an estimate of this coefficient can be obtained by identification methods if in at least one of the thermometric sections there is at least one supplementary thermometer favourably located;
- suitable numerical filters should be used to estimate, at each date of interest, the instantaneous rate of change of the thermometric readings.

6 – ACKNOWLEDGEMENTS

Heartfelt thanks are expressed to Guido MAZZA', of ENEL.HYDRO, for his useful comments and suggestions, and to Elia BON, likewise of ENEL.HYDRO, for his competent, effective and fast carrying out of all the numerical computations and graphic rendering of results.

7 (APPENDIX) – A POSSIBLE ALTERNATIVE APPROACH FOR THE CASE OF ARBITRARILY VARYING TEMPERATURES

As already stated, the above presented model may suffer from growing inaccuracies the more the yearly thermal cycle departs from a periodic one.
An alternative model exempt in principle from such limitations can be formulated (see FANELLI and GIUSEPPETTI, 1981) from the knowledge of the dam's 'natural cooling modes'. In the case of arch dams, again this model can be simplified thanks to the assumption of one dimensional heat flow at the generic elevation. The corresponding formula for the temperature across the generic horizontal section is then (the details are omitted for brevity):

$$\vartheta(x,t)=$$

$$=\left(1-\frac{x}{L}\right).\vartheta_{upst}(t)+\frac{x}{L}.\vartheta_{dwnst}(t)-\frac{2}{\pi}.\sum_{\tau\le t}\sum_{n=1,2,..}Exp\left(-a.\left(\frac{n.\pi}{L}\right)^2.(t-\tau)\right).\left[\begin{array}{l}\Delta\vartheta_{upst}(\tau).\sin\left(\frac{n.\pi.x}{L}\right)+\\ +(-1)^{n+1}.\Delta\vartheta_{dwnst}(\tau).\sin\left(\frac{n.\pi.x}{L}\right)\end{array}\right], \quad (7.1)$$

$\vartheta_{upst}(t)$, $\vartheta_{dwnst}(t)$ being the temperatures on the two faces at the elevation of the section, and $\Delta\vartheta_{upst}(\tau)$, $\Delta\vartheta_{dwnst}(\tau)$ the finite increments of the said temperatures over the finite time step $\Delta\tau$.
It is evident, then, that new 'monothermometric' influence coefficients $KM_{u,j}$, $KM_{d,j}$ can be computed, based on the temperature distributions:

$$\vartheta_{u,j}(\xi,\zeta)=-\frac{2}{\pi}.N_{m(j)}(\zeta).\sin(n.\pi.\xi), \quad , \quad \vartheta_{d,j}(\xi,\zeta)=\frac{2}{\pi}.(-1)^n.N_{m(j)}(\zeta).\sin(n.\pi.\xi) \quad (7.2)$$

(evidently $KM_{d,j}=(-1)^n.KM_{u,j}$), such that the thermal component of displacement will be given by:

$$\delta(t)=\delta_{st}(t)+\sum_{\tau\le t}\sum_{n=1,2,..}\sum_{j}KM_{u,j}.Exp\left(-a.\left(\frac{n.\pi}{L}\right)^2.(t-\tau)\right).\left[\Delta\theta_{upst}(\tau)+(-1)^n.\Delta\vartheta_{dwnst}(\tau)\right], \quad (7.3)$$

---

[8] The importance of an 'optimal' collocation of the thermometers is underlined by the fact that in the present model it was found necessary (see § 4) to identify *a posteriori* the relative proportions of the elevation-wise linear and parabolic components of the temperature distribution (as well as their time variations), which can be avoided by a judicious collocation of the instruments.

where $\delta_{st}(t)$ is the displacement that would be caused by a pseudo-stationary temperature distribution $\vartheta_{st}(t)=\sum_j N_{m(j)}(\zeta).[(1-\xi)\vartheta_{upst,j}(t)+\xi.\vartheta_{dwnt,j}(t)]$, and hence can in its turn be expressed as a linear function of the running upstream and downstream temperatures, the coefficients of this linear function being pre-computed monothermometric influence coefficients based on temperature distributions $N_{m(j)}(\zeta).(1-\xi)$, $N_{m(j)}(\zeta).\xi$.

The drawback of this more general approach is that in principle it requires to compute lengthy sums extended over the whole past thermal history, this necessity reflecting the physical fact that *stricto sensu* the thermal component of any structural effect in a massive structure is a *functional* of all past temperature values, rather than a *function* of the current values. (The requirement to effect a sum also over all the infinite integer, positive values of n is a less taxing one, thanks to the damping factor of the negative exponential, where n appears squared).

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Narx modeling of radial crest displacements of the Schlegeis Arch Dam

NARX MODELING OF RADIAL CREST DISPLACEMENTS OF THE SCHLEGEIS ARCH DAM

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ABSTRACT: Numerical models for the simulation of the static behaviour of the Schlegeis Arch Dam have been developed with reference to "Interpretation of measurements results – Theme C", formulated by Verbundplan. Two approaches have been pursued: namely, a classical statistical model has been compared to a nonlinear black-box model obtained with parametric identification techniques. In particular, a polynomial NARX (Non-linear Auto Regressive with eXogenous input) model has been employed. This model is shown to be better suited for prediction purposes of the benchmark with respect to the statistical model.
Key Words: statistical models, black-box identification, NARX models, prediction

1 INTRODUCTION

Static monitoring of dams is traditionally performed by measuring input (causes) and output (effects) variables, and correlating the data in a regression analysis framework. Usually, the role of cause variables is attributed to environmental conditions, such as the water level and the temperatures of the water, the air and specific points of the dam. On the other hand, displacements, mostly measured at points on the crest, and openings of contraction joints are chosen as output variables. Typically, static regression functions are determined which do not take the time variable into account: an istantaneous relationship is assumed between input and output variables. In other words, variations of an effect variable immediately follow the variation of a cause variable.

A more general approach would consider the dependency of output variables also on *past* values of the input variables, yielding a dynamical model. In this view, the dam-rock-reservoir system can be reasonably considered a dynamical model, though its evolution in time is much longer than what is usually experienced in the dynamics of structures.
To model the system's dynamic behaviour classical black-box identification techniques (Söderström and P. Stoica, 1988; Ljung, 1987; Johansson, 1993) can be successfully employed. In particular, nonlinear NARX (Non-linear Autoregressive with eXogenus input) models (Leontaritis and Billings, 1985a, 1985b) have been found to be extremely effective for similar applications, where both prediction and simulation accuracy is

required (see e.g. (Safak, 1991; Loh and Duh, 1996; Palumbo and Lancini, 1997a, 1997b; Pizzigalli, 1998; Palumbo and Piroddi, 2000, 2001)).

In this paper a polynomial NARX model has been identified on the Schlegeis Arch Dam data. It is shown that such a model is well suited for the prediction purposes of the benchmark.

2 STATISTICAL MODELS FOR STATIC MONITORING

Statistical modeling relies on the possibility of establishing a correlation between variables which act on the structure (cause or input variables) and the response of the structure to these excitations (effect or output variables). In the problem at hand, the radial crest displacement is assumed as the output variable, while the input variables are the water level and the temperatures measured at various points of the dam. The existing correlation between these variables is analyzed by means of a multiple linear regression such that the output variable is obtained as a linear combination of the input variables.

In the model, the hydrostatic ($f_{hyd}(t)$) and the thermic ($f_{th}(t)$) components appear separately:

$$y(t) = f_{hyd}(t) + f_{th}(t) . \tag{1}$$

The former is a polynomial of the water level, whereas the latter is defined by means either of a sinusoidal function or of a linear function of the temperatures measured at various points of the structure. The most accurate model found contains a 3[rd] order polynomial of the water level and a linear function of 5 out of the 6 available concrete temperature measurements:

$$f_{hyd}(t) = b_0 + b_1 W_{LEVEL}(t) + b_2 W_{LEVEL}(t)^2 + b_3 W_{LEVEL}(t)^3 , \tag{2a}$$

$$f_{th}(t) = c_1 T_{H12,UP}(t) + c_2 T_{H12,MI}(t) + c_3 T_{H12,DO}(t) + c_4 T_{H15,UP}(t) + c_5 T_{H15,DO}(t) , \tag{2b}$$

where the parameter values are reported in Table 1.

PAR.	VALUE	PAR.	VALUE
b_0	0.3930659 E+02	c_1	−0.3104975 E+01
b_1	0.2611623 E+02	c_2	−0.2878155 E+01
b_2	−0.2834200 E+02	c_3	−0.9622283 E+01
b_3	0.5850617 E+02	c_4	−0.1410552 E+01
		c_5	−0.1825318 E+02

Table 1: Parameter values of the static model

The model is characterized by the following statistical parameters: the correlation index R^2 has a value of 0.987 and the standard deviation of the residuals is equal to 1.434. On the whole, the model achieves a quite satisfactory level of accuracy with respect to the available data.

There are some important reasons why a static model may be considerate inadequate. In the first place, there is experimental evidence that at equal values of the water level the crest displacement depends not only on the value of the concrete temperature, but also on its derivative. In other words, there exists a dynamic effect which cannot be accounted for by a purely static model.
Secondly, the determination of the nonlinear structure of the model is largely the result of a trial-and-error procedure, which obviously cannot provide optimal results in any sense.
Finally, the model requires 5 temperature measurements, which may well be considered a redundant information (besides being costly).

3 THE ROLE OF DYNAMICS IN THE MODEL

The remarks at the end of the previous section point out the need to introduce dynamics in the model. Besides, the static model (2) is effective mostly because of the nonlinearity in the hydrostatic component. In view of this, a nonlinear dynamic modeling technique must be applied. In this paper NARX (Nonlinear AutoRegressive with eXogenous part) models are employed for this purpose (Leontaritis and Billings, 1985a, 1985b).

NARX models are the nonlinear generalization of the well known ARX models, which constitute a standard tool in linear black-box model identification (Ljung, 1987; Söderström and Stoica, 1988). These models can represent a wide variety of nonlinear dynamic behaviours and have been extensively used in various identification and control applications (see e.g. (Safak, 1991; Loh and Duh, 1996; Palumbo and Lancini, 1997a, 1997b; Pizzigalli, 1998; Palumbo and Piroddi, 2000, 2001)). NARX models are formulated as discrete-time input-output recursive equations:

$$y(t) = f(y(t-1), \ldots, y(t-n_y), u(t-1), \ldots, u(t-n_u)) , \tag{3}$$

where y and u are the output and input signals respectively, $f(\cdot)$ is a generic nonlinear function and n_y and n_u are the maximum delays for the y and u terms, respectively. Depending on how function $f(\cdot)$ is represented and parameterized, different NARX model structures and - consequently - identification algorithms are derived.

A particularly interesting class of NARX models is obtained when $f(\cdot)$ is a polynomial function of its arguments: in other words, $y(t)$ comes from a linear regression of terms (*regressors*) obtained as products of powers of past values of y and u. In fact, polynomial NARX models are *linear-in-the-parameters*, so that they can be identified with simple algorithms of the Least Squares family and relatively small effort.

4 THE NARX MODEL IDENTIFICATION PROCEDURE

Many efficient techniques have been introduced for the identification of polynomial NARX models (Korenberg *et al.*, 1987; Leontaritis and Billings, 1987; Billings *et al.*, 1989; Aguirre and Billings, 1995; Mao and Billings, 1997). Typically, an iterative procedure is used: at each step the model structure is modified by adding a regressor chosen in a given family with a convenient criterion, then the parameters of the augmented model are estimated by an identification method of the Least Squares (LS) family; the procedure ends when a specified level of accuracy is reached. Thus, the model structure and parameters are estimated *jointly*.

As is commonly done in the classical model identification framework, these algorithms select the optimal model with respect to *prediction* accuracy. This is unsuitable for our long range prediction application, where recent past values of the output variable (the radial crest displacement) are not available in the 1999 to 2000 period range. Therefore, a slightly modified identification algorithm will be employed here, which is based on the minimization of the *simulation* error, i.e. of the difference between $y(t)$ and its estimate computed on the basis of the *estimated* values of y and of the real values of u up to $t-1$.

In detail, the identification algorithm consists of the following steps:

1. **Initialization** - To initialize the procedure, it is necessary to provide a set of I/O data and the set of possible regressors that will be evaluated for inclusion in the model.
2. **Iteration (part 1: addition)** - Regressors not present in the current model are examined for possible inclusion. For each candidate regressor, the current model is augmented with it, the resulting model is estimated with Least Squares and evaluated *in simulation*. The first regressor is accepted which makes the simulation error variance decrease at least by a specified percentage with respect to the current model. If the candidate regressor set is exhausted, the best regressor found is accepted anyway.
3. **Iteration (part 2: pruning)** - When a regressor is added, an iterative pruning subprocedure is performed: for each model regressor, the submodel obtained by eliminating that regressor is estimated and evaluated. If the best reduced model still makes the simulation error variance decrease with respect to the previous major iteration, it becomes the current model. The subprocedure is repeated until no regressors can be eliminated without a performance loss with respect to the previous major iteration.
4. **Stop criterion** - The procedure is stopped when a specified accuracy is obtained or after a given maximum number of iterations.

Overall, the combined addition/pruning procedure aims at increasing accuracy while preserving (or even reducing) the number of parameters, thus enforcing the rationale that a parsimonious model is always preferable for robustness reasons. In fact, as the model complexity increases in terms of number of parameters its ability to explain the identification data also improves, but over a certain level of accuracy the robustness of the model starts to decrease, in the sense that the model becomes less and less generalizable to other data sets of the same system.

The standard deviation of the mean square simulation error

$$FIT = \sqrt{\frac{\sum_{t=1}^{N}[y(t) - y_{SIM}(t)]^2}{N}} \tag{4}$$

will be used in the sequel as a performance index for the evaluation of the model quality. In Eq. (4) N is the number of data samples, $y(t)$ and $y_{SIM}(t)$ are the measured output and the response calculated from the model in simulation mode, respectively.

5 EXPERIMENTAL RESULTS

Previous experience in static modeling of dams has shown that the input-output dynamics of the system can be described with good accuracy by means of NARX models with nonlinearities only in the exogenous part (Palumbo and Lancini, 1997b). In view of this, NARX models of this type with multiple inputs will be employed in the following.

A crucial step in the identification algorithm is the definition of the set of possible regressors that the algorithm must select for inclusion in the model. This set defines the family of possible model structures which can be considered by the identification algorithm. For obvious computational reasons, it is important to limit the dimension of this set. Previous experiences (Palumbo and Lancini, 1997b) together with the statistical model developed in Sect. 2 yield some useful indications for this task. In particular, nonlinear terms up to the 3rd order have been considered for the water level only and cross-terms involving different inputs or the input and output signals have been excluded. Also, various identification attempts with models of different structure have shown that no significant improvement of the *FIT* criterion is obtained if more than two inputs (the water level and one of the concrete temperatures) are used. In particular, the most effective temperature measurement was experimentally found to be $T_{H12,DO}$: the other temperature measurements are considered *redundant* for the aim of monitoring the dam.

Finally, for numerical reasons the mean has been removed from all the (linear and nonlinear) regressors: better simulation results have been obtained with the models identified with zero-mean regressors.

A 10 parameter NARX model has been identified. The maximum delay of the regressors is 2, so that the model defines a nonlinear 2nd order difference equation (1-DOF model)

$$y(t) = f(y(t-1), y(t-2), u_1(t-1), u_1(t-2), u_2(t-1), u_2(t-2)) \,, \tag{5}$$

where $f(\cdot)$ is a polynomial function of its arguments, $y(t)$ is the radial crest displacement, $u_1(t)$ is the water level and $u_2(t)$ is the temperature $T_{H12,DO}$. More precisely, the identified model has the following structure:

$$y(t) = a_1y(t-1) + a_2y(t-2) +$$
$$+ b_{11}u_1(t-1) + b_{12}u_1(t-2) + b_{21}u_1(t-1)^2 + b_{22}u_1(t-2)^2 + b_{31}u_1(t-1)^3 + b_{32}u_1(t-2)^3 +$$

$$+ c_1 u_2(t-1) + c_2 u_2(t-2) \,, \tag{6}$$

where the values of the coefficients a_i, b_{ij} and c_i are listed in Table 2. Notice that the hydrostatic and the thermic effects can still be separated. In fact, equation (6) can be viewed as a linear 2nd order model with two inputs, w_1 and w_2:

$$y(t) = a_1 y(t-1) + a_2 y(t-2) + w_1(t-1) + w_2(t-1) \,, \tag{7a}$$

where

$$w_1(t) = b_{11}u_1(t) + b_{12}u_1(t-1) + b_{21}u_1(t)^2 + b_{22}u_1(t-1)^2 + b_{31}u_1(t)^3 + b_{32}u_1(t-1)^3 \,, \tag{7b}$$

$$w_2(t) = c_1 u_2(t) + c_2 u_2(t-1) \,, \tag{7c}$$

to which the Principle of Superposition applies. Therefore, the hydrostatic effect is the solution of equation (7a) with $w_2(t) = 0$, while the thermic effect is correspondingly obtained with $w_1(t) = 0$. The overall output is simply the sum of these two solutions.

MODEL COEFFICIENT	ESTIMATED VALUE
a_1	1.060 E+00
a_2	−9.370 E−02
b_{11}	4.353 E−01
b_{12}	−4.124 E−01
b_{21}	4.825 E−03
b_{22}	−4.538 E−03
b_{31}	1.861 E−05
b_{32}	−1.702 E−05
c_1	−1.535 E−01
c_2	8.692 E−02

Table 2: Regressors and estimated parameter values of NARX model

The overall performance of the model is summarized by a value of the *FIT* index of 1.493. The model accuracy in simulation can be appreciated in Fig. 1.

As already pointed out, the model estimations and the resulting simulations have all been performed using zero-mean signals, i.e. the mean value (computed on the first 2557 data) has been subtracted to $y(t)$, $u_1(t)$, $u_1(t)^2$, $u_1(t)^3$ and $u_2(t)$ in equation (6). Therefore, to obtain comparable values of the model outputs it is necessary to add *a posteriori* a value of 45.8625 mm to $y(t)$ (clearly, this operation must be repeated also for the predicted values relative to the time period 1999-2000).

If the maximum delay associated to the exogenous terms in the NARX model is increased, i.e. longer time dependencies are allowed in the model, even more accurate models can be obtained, at the cost of an increased size of the model and of a longer identification procedure. Similar results can be obtained if cross-terms are allowed to appear in the regressor set which defines the possible model structures.

However, the improvements in terms of the *FIT* criterion are not so significant to justify the model dimension increase. In fact, as already discussed, as the model complexity increases, the identified model will reproduce all the more accurately the data used for the identification itself while being less capable of generalizing to alternative data sets. In this application, where a long term prediction accuracy is requested, the robustness issue is crucial and this phenomenon (usually termed *overfitting*) must be prevented at all costs.

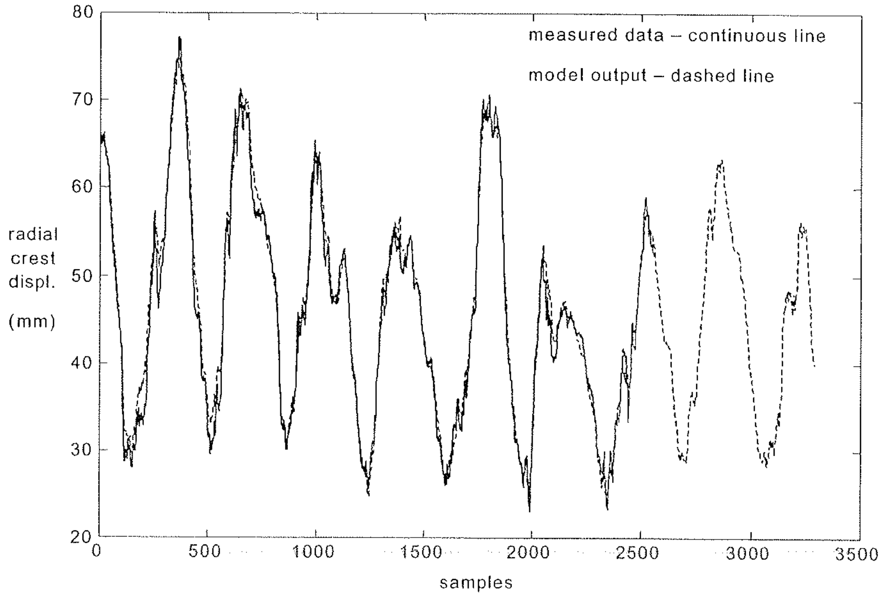

Figure 1: Simulation performance of NARX model

In the proposed identification algorithm the growth of the model complexity is somewhat limited by the pruning phase. However, a more systematic way to prevent the occurrence of overfitting can be applied if the available data are divided in two sub-sets: the identification data set and the validation data set. Then, the first data set only is used for the estimation of parameters, while the second one is used to monitor the identified model performance on data not used in the estimation phase. In this way, the identification procedure may be stopped when the selected model achieves the best average performance on both identification and validation data.

To test the validity of the model structure found by the identification algorithm the following test has been performed. The model parameters have been re-estimated using only the first 90% of the available data, and the performance of the identified model has been measured on the remaining 10% of data: apparently, the model accuracy does not degrade in the last part of the data set. Indeed, the value of the $FIT_{10\%}$ index obtained in the last 10% of the data is equal to 1.218, which is comparable to - and even better than - what obtained in the sub-set of the data set used for the identification phase ($FIT_{90\%}$ = 1.477). This shows that the model structure selected by the identification algorithm is indeed *robust*, i.e. capable of generalizing to data not used for parameter estimation. This is a good clue that the model can be confidently used for long range prediction.

6 CONCLUSIONS

Two numerical models for the simulation of the static behaviour of the Schlegeis Arch Dam have been developed following two different rationales. A classical statistical approach was used to derive a 9-parameter model, where the hydrostatic and the thermic components appear separately. The former is obtained as a 3^{rd} order polynomial of the water level, whereas the latter is a linear combination of 5 concrete temperature measurements.

A dynamical NARX model has also been developed using a novel identification algorithm. The model nonlinear structure is selected automatically by the algorithm, which explores a set of possible model structures defined by the user also on the light of what obtained with the statistical approach. The robustness and generalization capability of the NARX model are demonstrated experimentally.

The NARX model requires 10 parameters, but only one temperature measurement. The hydrostatic and thermic effects can still be computed separately.

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