

# NONLINEAR ANALYSES OF EMBANKMENT DAMS UNDER STRONG EARTHQUAKE

Tsdatsugu Tanaka

*Professor Emeritus of the University of Tokyo  
President, The Japan Association of Rural Resource  
Recycling Solutions, Tokyo, Japan*

# OUTLINE

## 1 INTRODUCTION

## 2 MATERIAL MODEL FOR CYCLIC BEHAVIOR OF SOIL

*2.1 Elasto-plastic model*

*2.2 Kinematic hardening model*

## 3 SHAKING TABLE TESTS OF EMBANKMENT DAMS AND DYNAMIC ANALYSES

*3.1 Outline of a model dam experiment*

*3.2 Dynamic analysis of embankment dam by simple constitutive model*

## 4 ARATOZAWA & OOGAKI DAM ELASTO-PLASTIC DYNAMIC ANALYSIS

*4.1 Aratozawa Dam dynamic analysis*

*4.2 Oogaki dam dynamic*

## 5 SUMMARY

# INTRODUCTION

Simple strain softening material model for soil is used with the features of non-associated flow characteristics, post-peak strain softening, and strain-localization into a shear band. Then a kinematic hardening model considering the cumulative deformation by cyclic loading is developed based on the soil model of isotropic strain-hardening-softening property.

Total stress elasto-plastic constitutive model is rather simple and robust for application to a dynamic response analysis of fill-type dams. A cumulative damage concept for simple elasto-plastic model is effective by using the results of cyclic tri-axial tests of saturated soils.

Dynamic progressive failure analysis of a small dry sand dam on shaking table is carried out. The computed acceleration and displacement at the crest of model dam is compared to the measured one. The computation of real rockfill dam is also carried out by total stress elasto-plastic model and effective stress constitutive model by taking into account the pore water build-up.

# YIELD & PLASTIC POTENTIAL FUNCTION

The yield function ( $f$ ) and the plastic potential function ( $\Phi$ ) are given by

$$f = \alpha I_1 + \frac{\bar{\sigma}}{g(\theta_L)} - K = 0$$

$$K = \frac{6c \cos \phi}{\sqrt{3}(3 - \sin \phi)}$$

$$\Phi = \alpha' I_1 + \bar{\sigma} - K = 0$$

$$\alpha = \frac{2 \sin \phi}{\sqrt{3}(3 - \sin \phi)}$$

$$\alpha' = \frac{2 \sin \psi}{\sqrt{3}(3 - \sin \psi)}$$

$I_1$  : primary invariant of stress

$\bar{\sigma}$  : second invariant of deviatoric stress

$$\frac{\sigma_1}{\sigma_3} = -K_{Rowe} \left( \frac{d\varepsilon_1^p}{d\varepsilon_3^p} \right)$$

# YIELD FUNCTION

In case of Mohr-Coulomb model,  $g(\theta_L)$  takes the following form

$$g(\theta) = \frac{3 - \sin \phi}{2\sqrt{3} \cos \theta - 2 \sin \theta \sin \phi}$$

$\phi$  : mobilized friction angle

$\theta$  : Lode angle (third invariant of deviatoric stress)

# SIMPLE STRAIN SOFTENING CONSTITUTIVE MODEL

Frictional softening is given by next function

$$\alpha(\kappa) = \alpha_p + \frac{\alpha_1 \kappa}{B + \kappa} \quad \alpha_1 = -(\alpha_p - \alpha_R)$$

Cohesion softening is given by next function.

$$K(\kappa) = K_p + \frac{K_1 \kappa}{D + \kappa} \quad K_1 = -(K_p - K_R)$$

Dilatancy is reduced by next equation

$$\alpha'(\kappa) = \alpha'_p \left(1 - \frac{\kappa}{C + \kappa}\right)$$

$\kappa$  is plastic parameter,  $B$ ,  $C$ ,  $D$  are constants for softening function  
 $p$  and  $R$  specify peak and residual

# ELASTIC PROPERTIES

Shear modulus **G**, Bulk modulus **K** and damping ratio **h** are given by Hardin-Drnevich equation

$$G = \frac{G_0}{1 + \frac{\gamma}{\gamma_r}}$$

$$h = \frac{\frac{\gamma}{\gamma_r}}{1 + \frac{\gamma}{\gamma_r}} h_{\max}$$

$$G_0 = G_E \frac{(2.17 - e)^2}{1 + e} \sigma_m^{0.4}$$

$$K = \frac{2(1 + \nu)}{3(1 - 2\nu)} G$$

$\gamma$  is shear strain,  $\gamma_r$  is reference shear strain,  $\nu$  is Poisson's ratio,  $e$  is void ratio and,  $G_E$ ,  $h_{\max}$  are empirical constants

# FRICTIONAL HARDENING-SOFTENING FUNCTIONS IN ISOTROPIC HARDENING MODEL

Hardening regime  $(\kappa \leq \varepsilon_f)$

$$\alpha(\kappa) = \left( 2 \frac{\sqrt{\kappa \varepsilon_f}}{\kappa + \varepsilon_f} \right)^m \alpha_p$$

Softening regime  $(\kappa > \varepsilon_f)$

$$\alpha(\kappa) = \alpha_R + (\alpha_p - \alpha_R) \exp \left\{ - \left( \frac{\kappa - \varepsilon_f}{\varepsilon_r} \right)^2 \right\}$$

$$\alpha = \frac{2 \sin \phi}{\sqrt{3}(3 - \sin \phi)}$$

$$\alpha' = \frac{2 \sin \psi}{\sqrt{3}(3 - \sin \psi)}$$

where  $m$ ,  $\varepsilon_f$  and  $\varepsilon_r$  are material constants



# PEAK FRICTION ANGLE

The peak friction angle ( $\phi_p$ ) is estimated from the empirical relations (by Tatsuoka)

$$\phi_p (\text{deg}) = \left\{ 59.47(1.5 - e) - 10(1 - e) \log \left\{ \frac{\sigma_3}{(\sigma_3)_0} \right\} \right\} g_R(\delta)$$

$$(\sigma_3)_0 = 4(1 - e)p_a (p_a = 98kPa)$$

$e$  : Initial void ratio

$\delta$  : angle of  $\sigma_1$  direction relative to horizontal bedding plane

# RETURN-MAPPING ALGORITHM

A change in stresses can cause an associated change in the elastic strains even by

$$d\boldsymbol{\varepsilon}^e = [\mathbf{c}]^{-1} (\boldsymbol{\sigma}_B - \boldsymbol{\sigma}_A) \quad [\mathbf{C}]: \text{Elastic matrix}$$

As the total strain does not change during the relaxation process, the plastic strain change is balanced by an equal and opposite change in the elastic strains :

$$d\boldsymbol{\varepsilon}_{ij} = d\boldsymbol{\varepsilon}_{ij}^e + s d\boldsymbol{\varepsilon}_{ij}^p$$

$$s d\boldsymbol{\varepsilon}_p = -d\boldsymbol{\varepsilon}^e = -[\mathbf{c}]^{-1} (\boldsymbol{\sigma}_B - \boldsymbol{\sigma}_A)$$

$$s = F_b / F_e \quad F_e \text{ is the area of the element}$$

$F_b$  is the area of a single shear band in each element

# RETURN-MAPPING ALGORITHM(continued)

$$d\epsilon^p = \lambda (\partial\Phi / \partial\sigma)$$

The plastic strain increments are proportional to the gradient of the plastic potential

$$\sigma_B = \sigma_A - S\lambda[D]^e (\partial\Phi / \partial\sigma)$$

Suitably relaxed stress must satisfy the yield function

$$f(\sigma_B, \kappa_B) = f\{(\sigma_A - S\lambda[D]^e (\partial\Phi / \partial\sigma)), (\kappa_A + \lambda)\} = 0$$

$$\lambda = f(\sigma_A, \kappa_A) / \{S(\partial f / \partial\sigma)[D]^e (\partial\Phi / \partial\sigma) - (\partial f / \partial\kappa)\}$$

# EXPLICIT DYNAMIC (RELAXATION) METHOD

$$M_D a + C v + P - P^{init} = F \quad (1)$$

$$C = \alpha M_D \quad (2)$$

$$q_{n+1} = \frac{1}{1 + 0.5\alpha\Delta t} \left[ \frac{\Delta t^2}{M_D} (F - P + P^{init})^t + 2q_n - (1 - 0.5\Delta t)q_{n-1} \right] \quad (3)$$

$M_D$  is the diagonalized mass matrix

$C$  is the damping matrix

$a$  is the acceleration vector

$q_n$  is the displacement vector at time  $n$ ,

$P$  is the internal force vector

$v$  is the velocity vector

$\Delta t$  is the time increment

$\alpha$  is the critical damping ratio

# EXPLICIT DYNAMIC (RELAXATION) METHOD

$$\alpha = \sqrt[2]{\frac{x^T K x}{x^T M^t x}} \quad (1)$$

$$\alpha = \sqrt[2]{\frac{q^T K^{lt} q}{q^T M_D q}} \quad (2)$$

$$K^{lt} = \frac{P^t - P^{t-\Delta t}}{\Delta t^t \nu} \quad (3)$$

$$\Delta t \leq \beta \frac{l}{V_c} \quad (4)$$

$$\Delta t \leq \beta \sqrt{\frac{\rho(1+\nu)(1-2\nu)}{E(1-\nu)}} \quad (5)$$

# IMPLICIT-EXPLICIT DYNAMIC (RELAXATION) METHOD

$$\tilde{q}_{n+1} = q_n + \Delta t v_n + \Delta t^2 (1 - 2\beta) a_n / 2$$

$$\tilde{v}_{n+1} = v_n + \Delta t (1 - \gamma) a_n$$

$v_n$  is the velocity vector;  $a_n$  is the acceleration vector at time  $n$ , and  $\gamma, \beta$  are constants

$$K^* = M / (\Delta t^2 \beta) + \gamma C_T / (\Delta t \beta) + K_T(\tilde{q}_{n+1})$$

$$K^* = M / (\Delta t^2 \beta)$$

$$K^* \Delta q = \Psi$$

Displacement is simultaneously solved from the explicit and implicit effective stiffness matrix using the Skyline solver

# IMPLICIT-EXPLICIT DYNAMIC (RELAXATION) MRTHOD

The residual force is evaluated by the equation.

$$\Psi = f_{n+1} - Ma_{n+1} - p(\tilde{q}_{n+1}, \tilde{v}_{n+1})$$

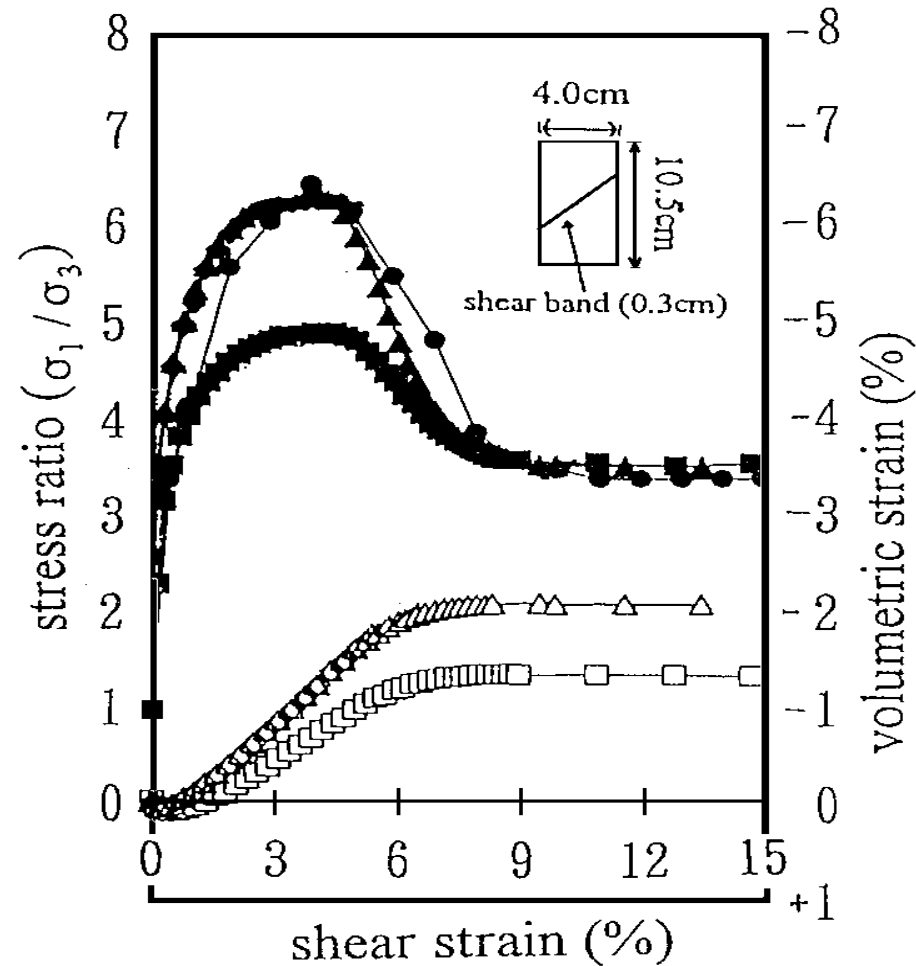
The displacement, velocity and acceleration of the next step are calculated by the following equations.

$$q_{n+1} = \tilde{q}_{n+1} + \Delta q$$

$$v_{n+1} = \tilde{v}_{n+1} = \Delta t \gamma a_{n+1}$$

$$a_{n+1} = (q_{n+1} - \tilde{q}_{n+1}) / (\Delta t^2 \beta)$$

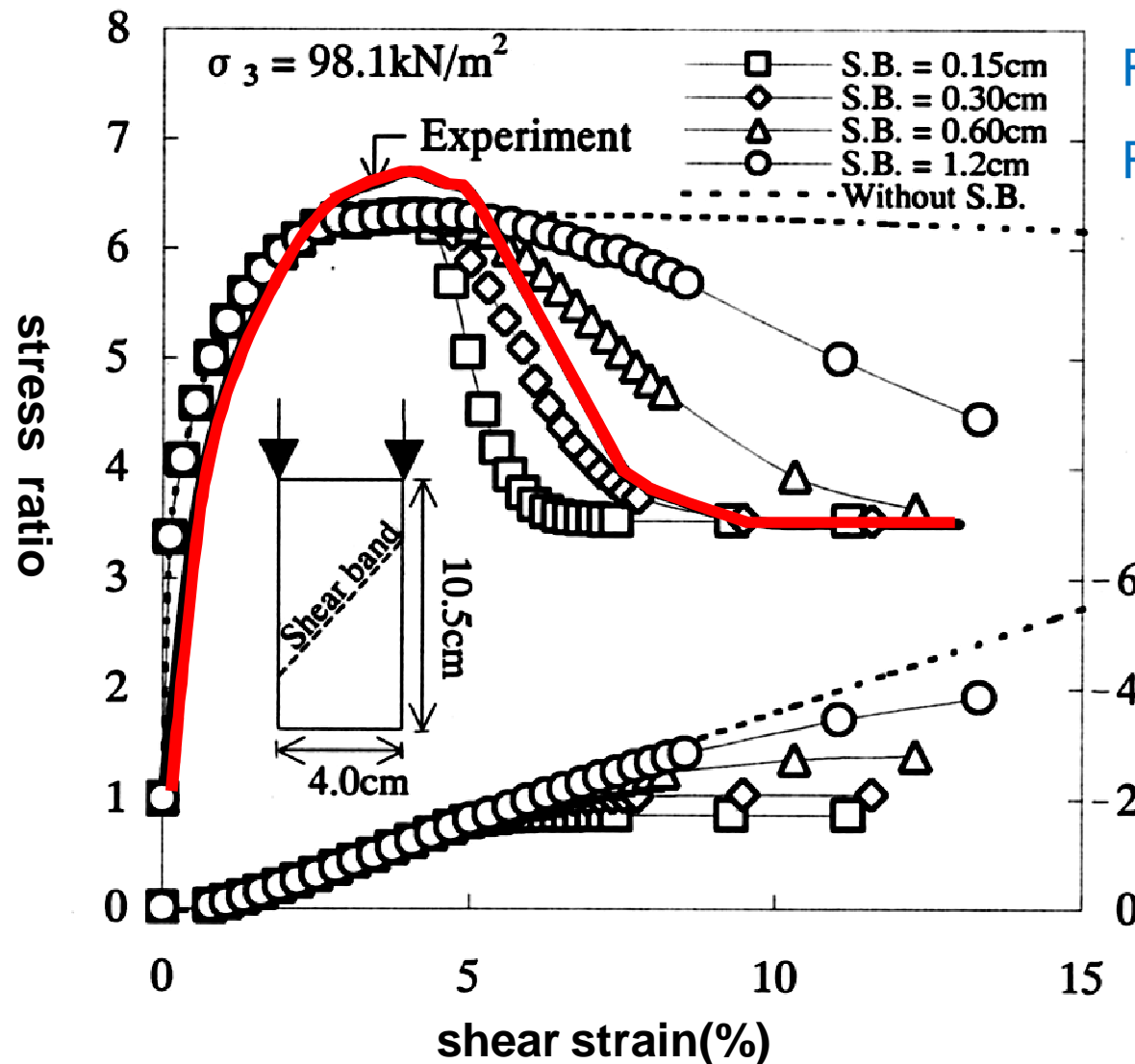
# PLANE STRAIN TEST & FINITE ELEMENT ANALYSIS



	Stress ratio	Volumetric Strain
Experiment( $\sigma_3=98.1\text{kN/m}^2$ )	●	—
Analysis( $\sigma_3=98.1\text{kN/m}^2$ , S.B.=0.3cm)	▲	△
Analysis( $\sigma_3=392.4\text{kN/m}^2$ , S.B.=0.3cm)	■	□



# PLANE STRAIN TEST & FINITE ELEMENT ANALYSIS USING 1 ELEMENT



Relative Density : 85%

Residual Friction Angle: 34°

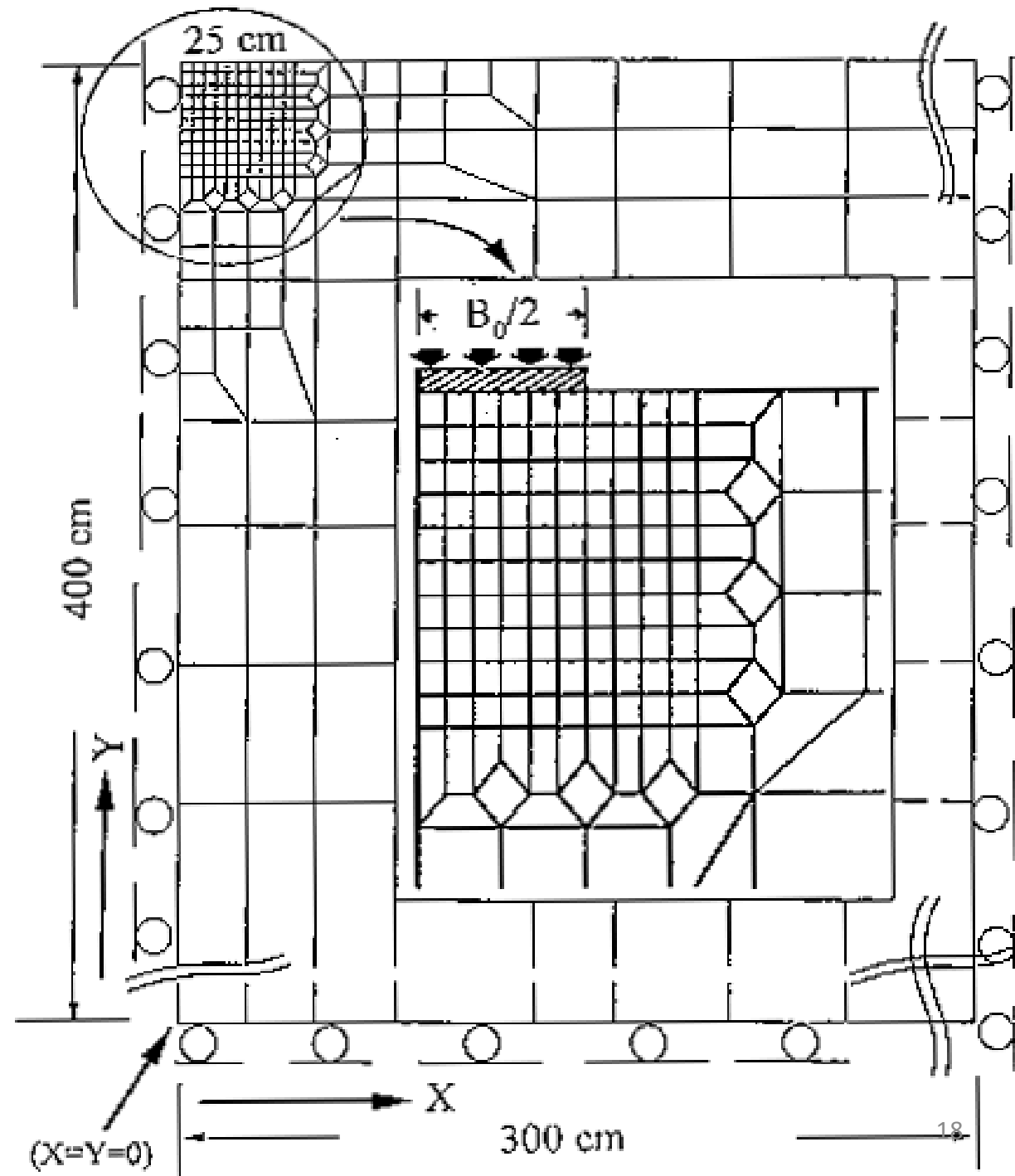
Volumetric strain (%)

Thickness of Shear Zone is 0.3 cm

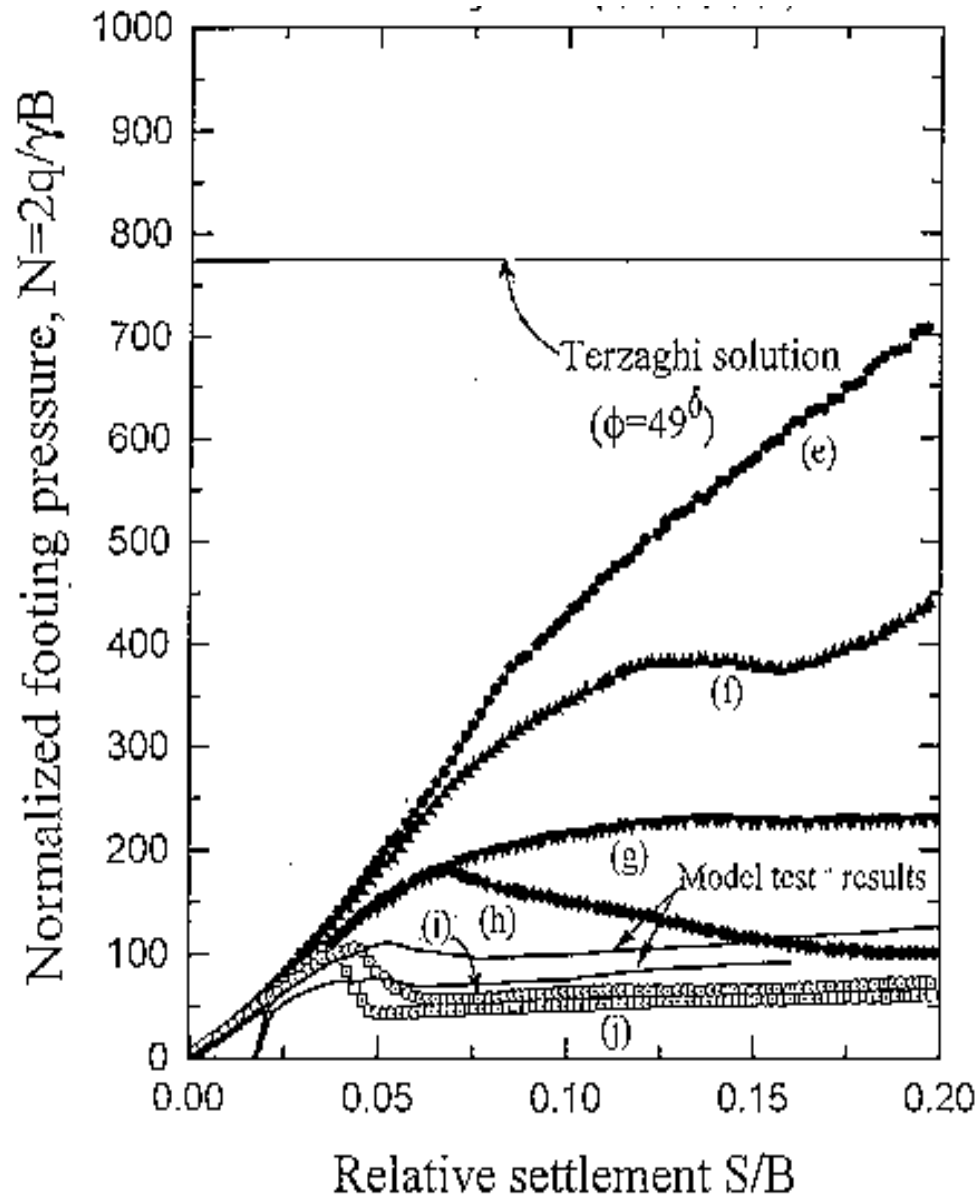
Plane Strain Test by Tatsuoka et. al

# FINITE ELEMENT MESH FOR FOOTING

Soils and Foundations Vol. 39,  
N0.4, 93-109, Aug. 1999

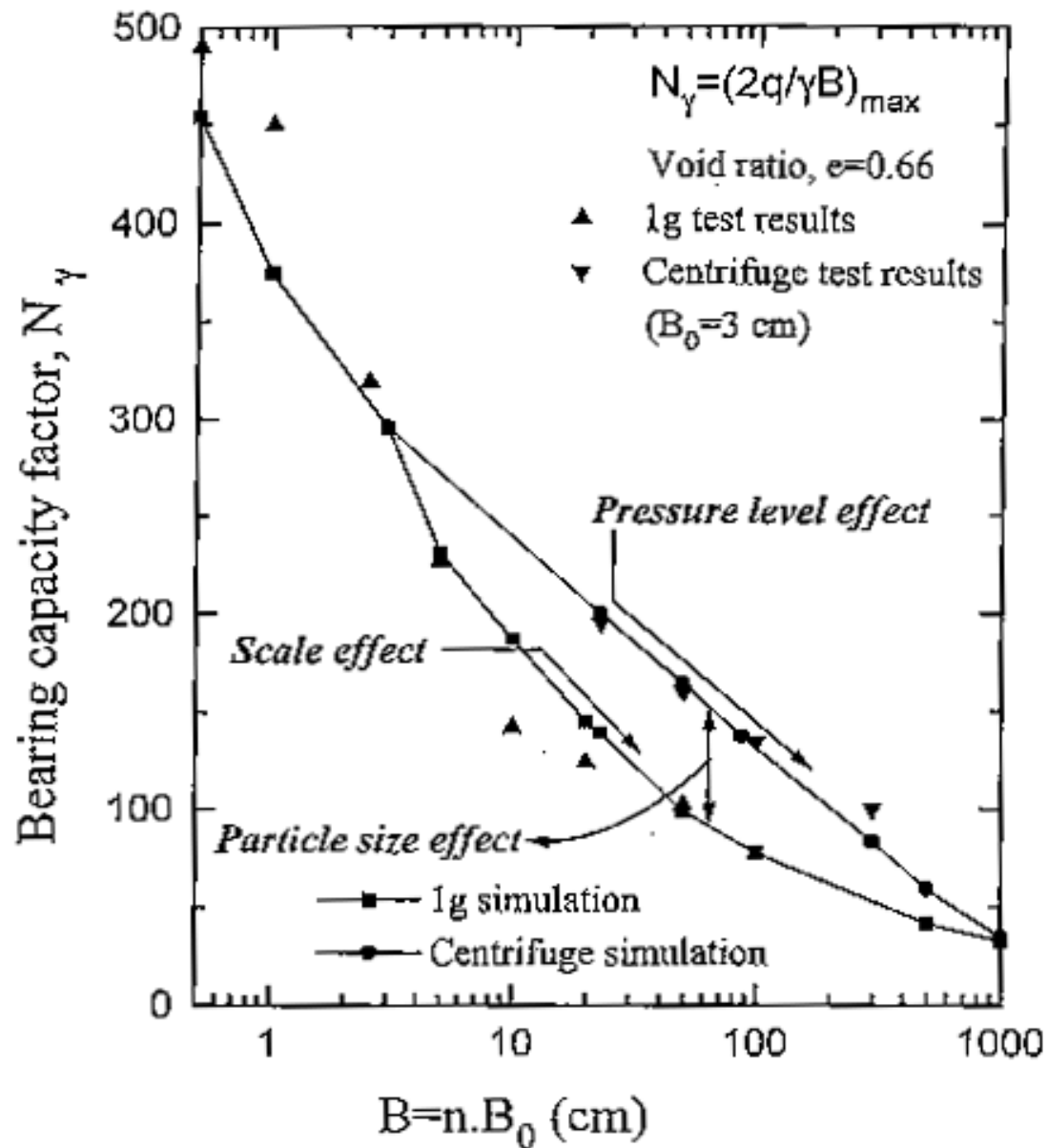


# RELATIONSHIPS BETWEEN NORMALIZED FOOTING PRESSURE AND RELATIVE DISPLACEMENT



- e: friction is assumed isotropic
- f : pressure level dependency
- g: anisotropic and pressure level
- h: anisotropic, pressure level and strain softening
- i: anisotropic, pressure level, strain softening and shear band

# RELATIONSHIP BETWEEN BEARING CAPACITY AND WIDTH SCALE EFFECT



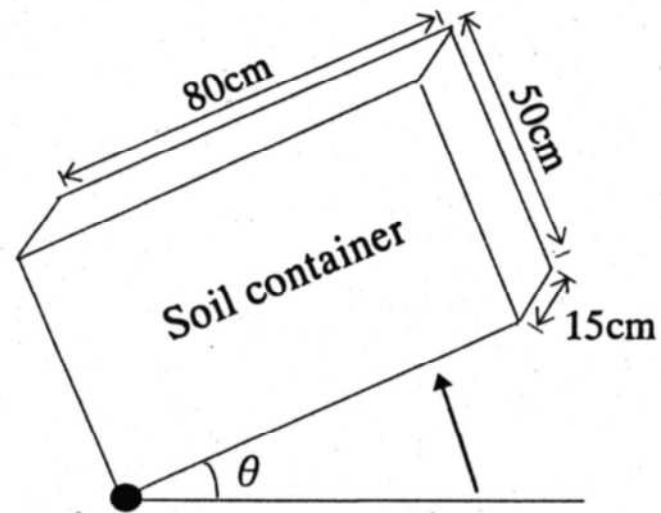
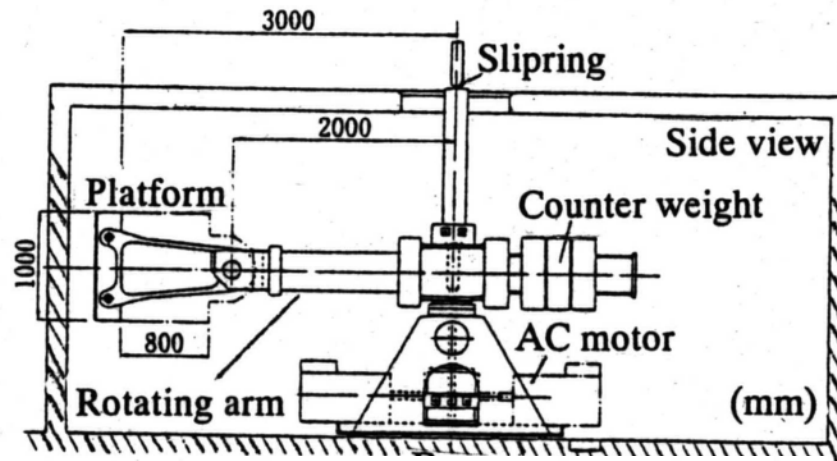
# BEARING CAPACITY COEFFICIENT

- FEM simulates closely the pre-peak behavior and peak load observed
- Simulates very well the bearing capacity in both types physical tests (1g and centrifuge condition)
- Can simulate not only the pressure level effect but also the particle size effect (shear band effect)

# **CENTRIFUGE STATIC TILTING MODEL TEST AND FINITE ELEMENT ANALYSIS**

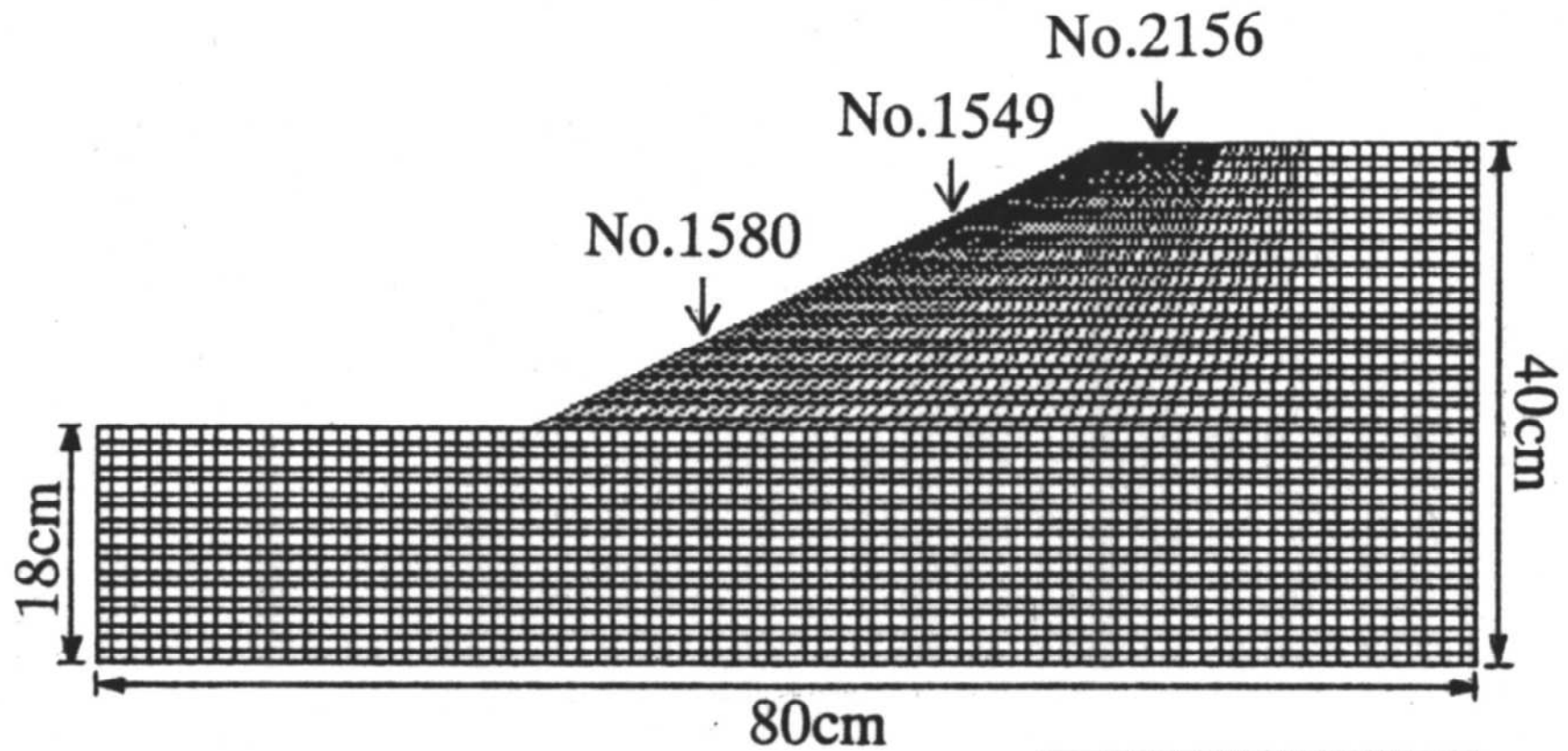
- **The pseudo-static model test under centrifuge acceleration 50g was carried out**
- **The horizontal seismic force was applied by tilting the table**
- **The soil for this test was a mixture of Toyoura sand and kaolin**

# CENTRIFUGE TEST SIMULATOR



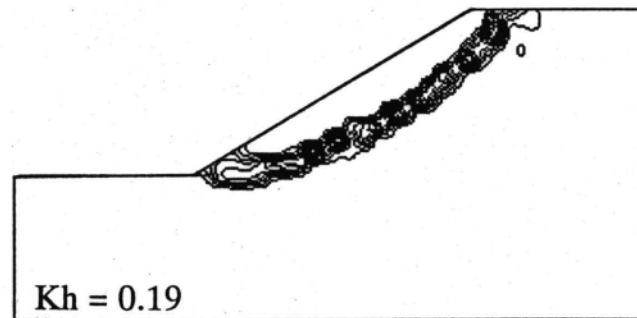
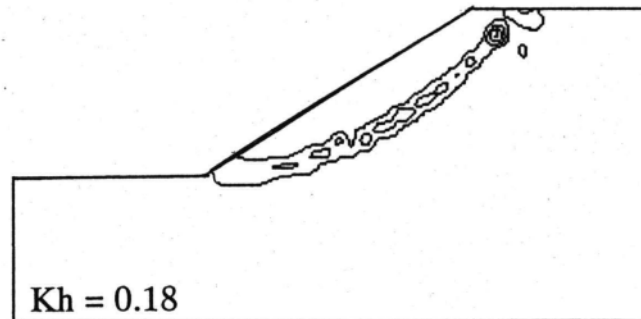
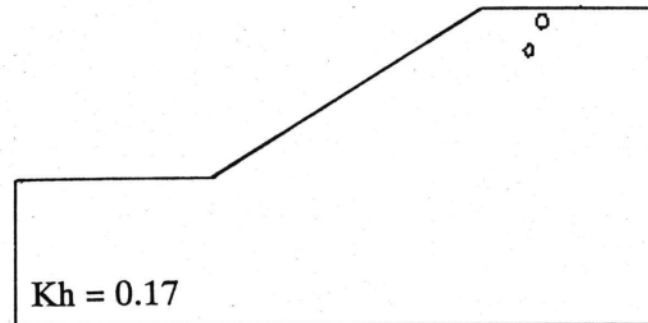
Tilting of soil container

# FINITE ELEMENT MESH FOR CENTRIFUGE STATIC TILTING MODEL TEST





# MAXIMUM SHEAR STRAINS BY FINITE ELEMENT ANALYSES

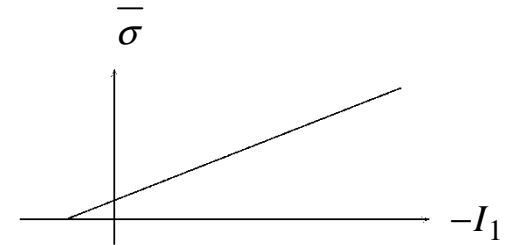


Interval of 5%

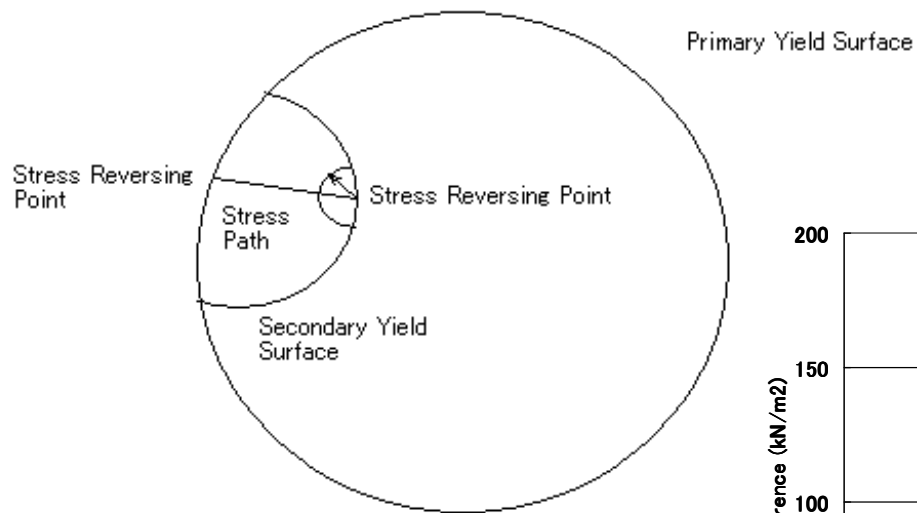
$K_h$ : Seismic coefficient

These are considered to be a kind of push-over analysis

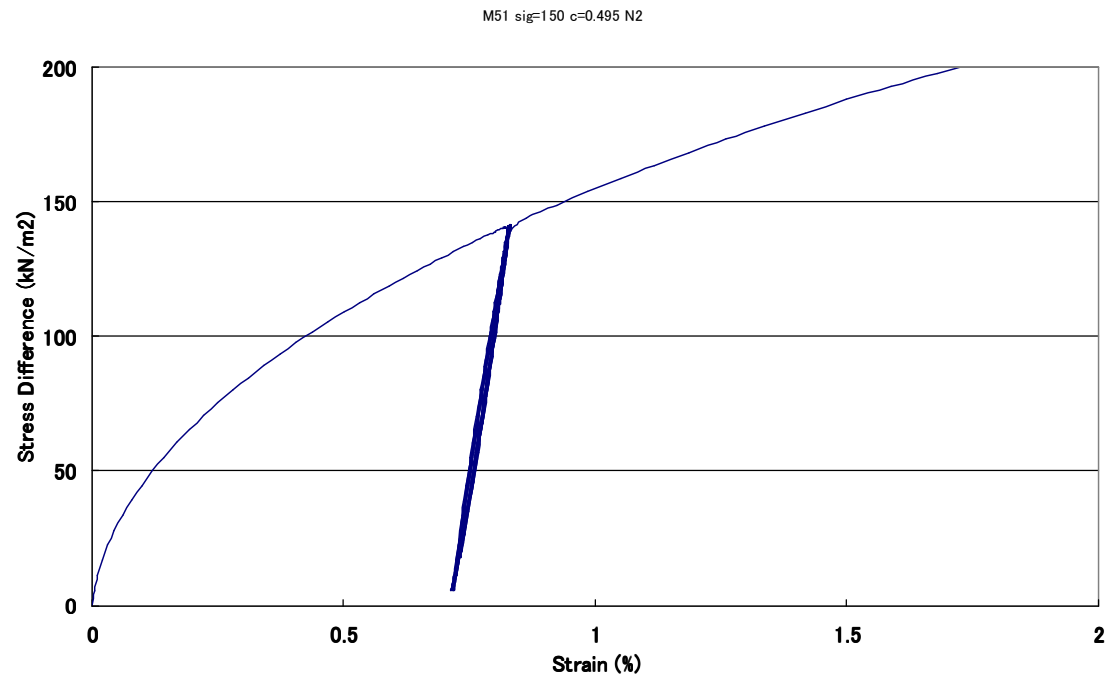
# KINEMATIC HARDENING CONSTITUTIVE MODEL



$-I_1$  and  $\bar{\sigma}$  relation



Mohr-Coulomb model takes pyramid shape in plane



Calculated stress strain relation of tri-axial test

# KINEMATIC HARDENING MODEL WITHIN BOUNDING SURFACE

$$\alpha_{iy}(\kappa') = \left( \frac{2\kappa'^l (\varepsilon_f / a_f)^l}{\kappa'^{2l} + (\varepsilon_f / a_f)^{2l}} \right)^m \alpha_p$$

$$\alpha(\kappa') = \alpha_{iy}(\kappa')(1 + e^n), \quad e = \eta / R$$

$$\alpha_{id}(\kappa') = \alpha - \alpha_p(e - 1.0) \quad e \geq 1.0$$

$$\alpha_{id}(\kappa') = \alpha \quad e < 1.0$$

$$\sin \psi' = \frac{3\sqrt{3}\alpha_{id}}{2 + \sqrt{3}\alpha_{id}}$$

# STRENGTH REDUCTION IN TOTAL STRESS ANALYSIS

- The advantage of the total stress analysis is that it is simple and numerically stable. The strength reduction in total stress is due to the plasticity or damage from a viewpoint of effective stress, cumulative shear strain that is similar to equivalent plastic parameter can be calculated in the elastic state.

The integral of shear strain increments can be given by next equation.

$$\bar{\varepsilon} = \int d\bar{\varepsilon} \quad d\bar{\varepsilon} = (de_x^2 + de_y^2 + de_z^2) + 2de_{xy}^2$$

where  $de_x, de_y, de_z, de_{xy}$  are deviatoric components of strain.

- By applying empirical factor to this value by using the cyclic tri-axial test result, we can estimate the reduction of strength in total stress analysis.

# SOLUTION OF (DYNAMIC) ELASTO-PLASTIC FINITE ELEMENT ANALYSIS

## Element type

Very few element types can avoid the shear locking and dilatancy locking

- One point integration of 4 nodes iso-parametric element with hour-glass control ( 8 nodes in three dimension element)
- 15 nodes triangular element (PLAXIS)

## Nonlinear solution method to avoid the accumulation of error

Dynamic equilibrium iteration is absolutely necessary (implicit method)

Return mapping method by explicit method is effective

## Strain softening with shear banding

Objectivity of analysis (mesh independancy) : incorporating a characteristics length of shear band in the material modeling based on physical experimental observations

## Simple check

Static footing limit load analysis and pseudo-static slope analysis are good benchmarks regarding above remarks

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*3.2 Dynamic analysis of embankment dam by simple constitutive model*

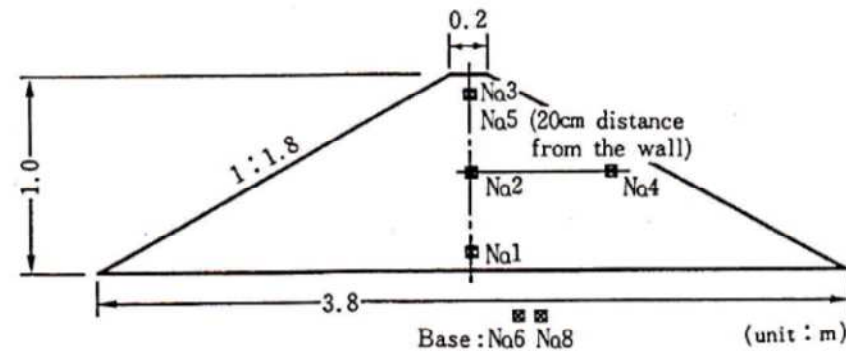
## 4 ARATOZAWA & OOGAKI DAM ELASTO-PLASTIC DYNAMIC ANALYSIS

*4.1 Aratozawa Dam dynamic analysis*

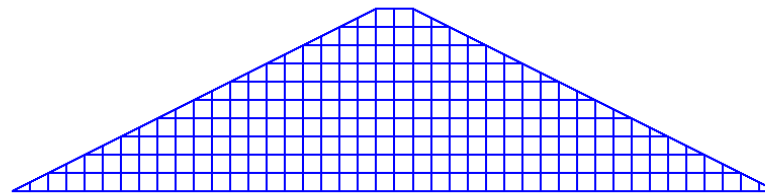
*4.2 Oogaki dam dynamic*

## 5 SUMMARY

# EMBANKMENT DAM MODEL AND LOCATION OF ACCELEROMETERS

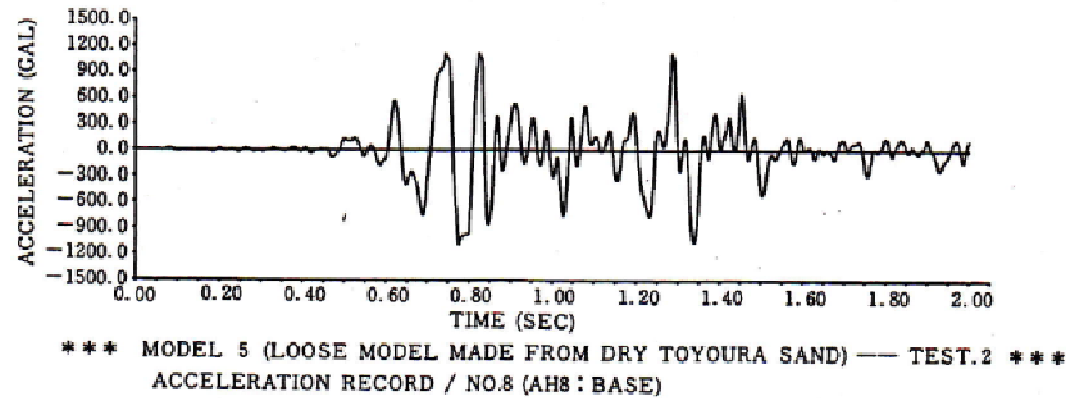


Embankment dam model and the location of accelerometers

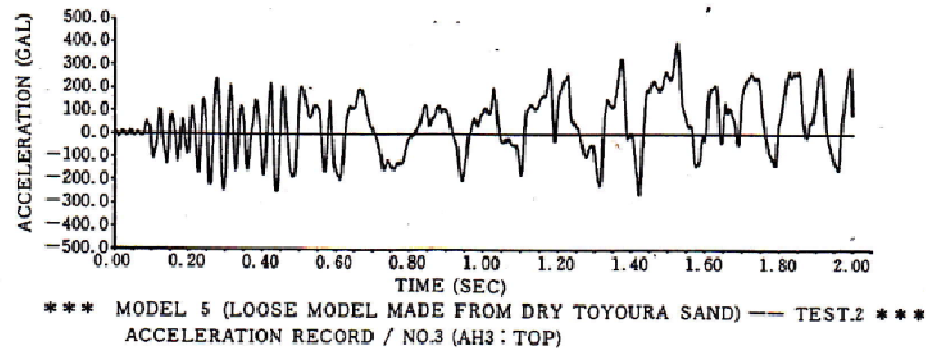


Finite element mesh used for the analysis

# OBSERVED ACCELERATION AT THE BASE AND CREST OF DAM MODEL



Input horizontal acceleration (observed at the base of shaking table)

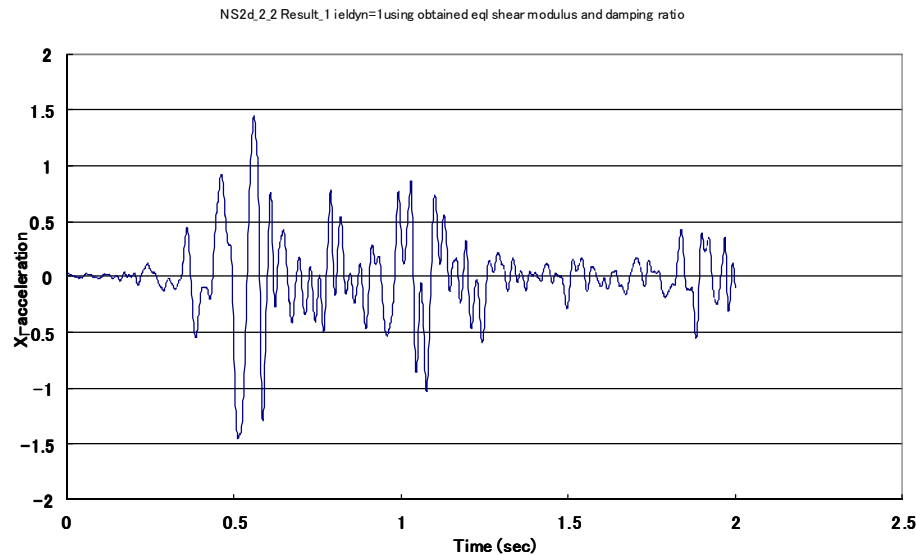


Observed acceleration at crest of the dam

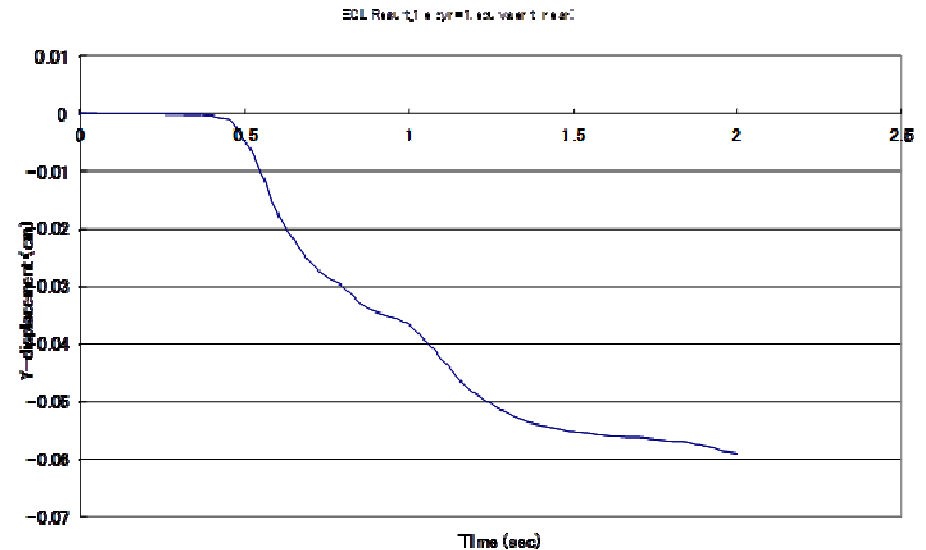


# COMPUTED ACCELERATION AND SETTLEMENT (1)

Shear modulus and damping ratio are estimated by applying the equivalent linear method  
dry density =  $0.0014 \text{ kg/cm}^3$ ,  $\phi_P = 35^\circ$ ,  $\phi_R = 34^\circ$ ,  $G_E = 1200.0$ ,  $h_{\max} = 0.25$ .



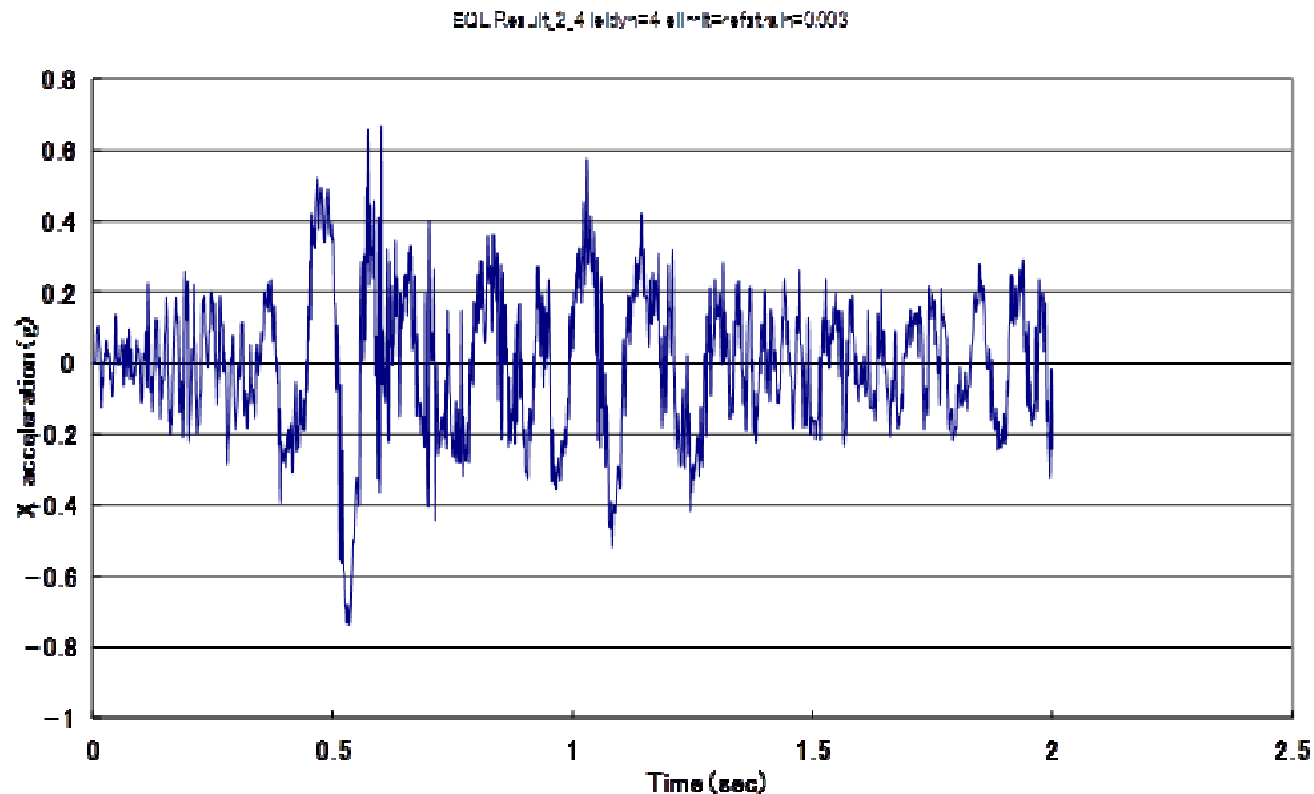
Computed horizontal acceleration  
at the center of dam crest



Computed settlement at the center of dam  
crest

## COMPUTED ACCELERATION (2)

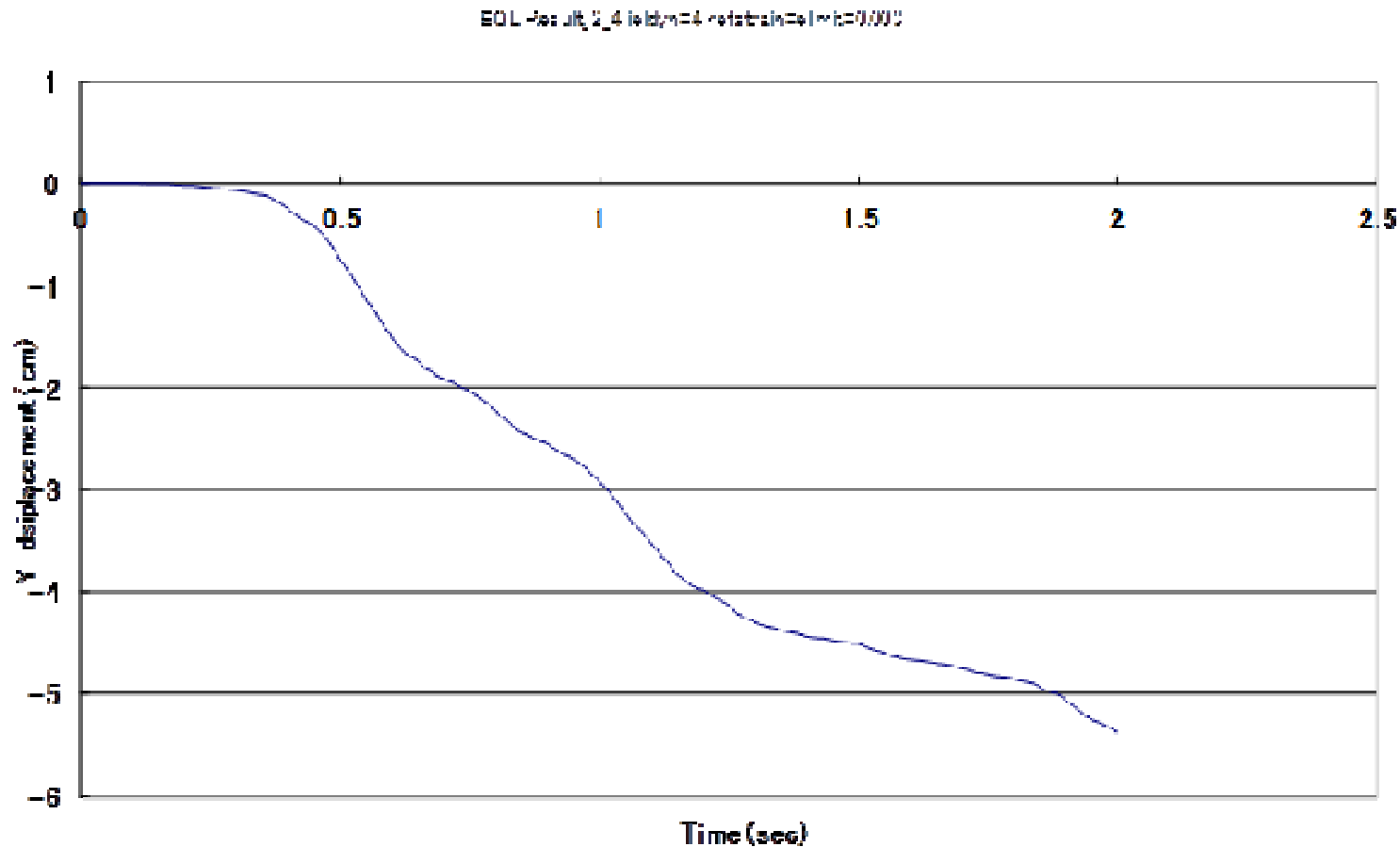
### ELASTIC LIMIT: reference shear strain for Hardin-Drnevich equation



Computed acceleration at the center of dam crest.  
Rayleigh damping  $\beta = 0$ , elastic limit = 0.0003

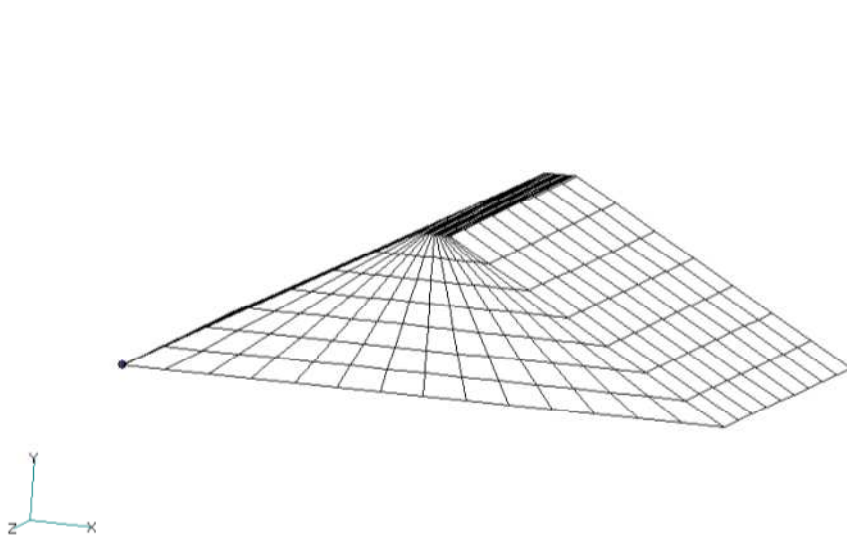
# COMPUTED SETTLEMENT(2)

NONLINEAR ELASTIC LIMIT: reference shear strain for  
Hardin-Drnevich equation

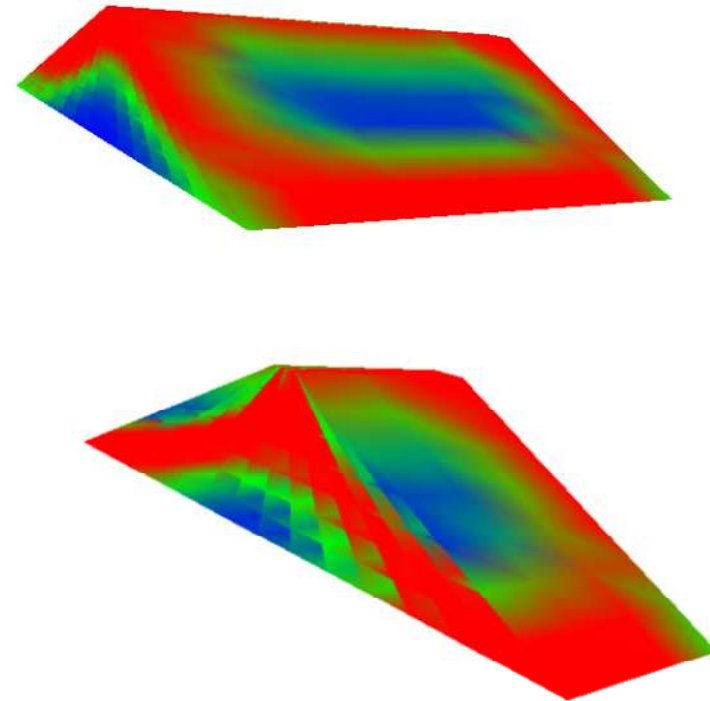


Computed settlement at the center of dam crest.  
Rayleigh damping  $\beta = 0$ , elastic limit = 0.0003

# THREE DIMENSIONAL FINITE ELEMENT ANALYSIS OF MODEL DAM ON SHAKING TABLE TEST



Three dimensional finite element mesh



Peak maximum shear strain 29%

Computed maximum shear strain of model dam  
after shaking (peak strain 30%)

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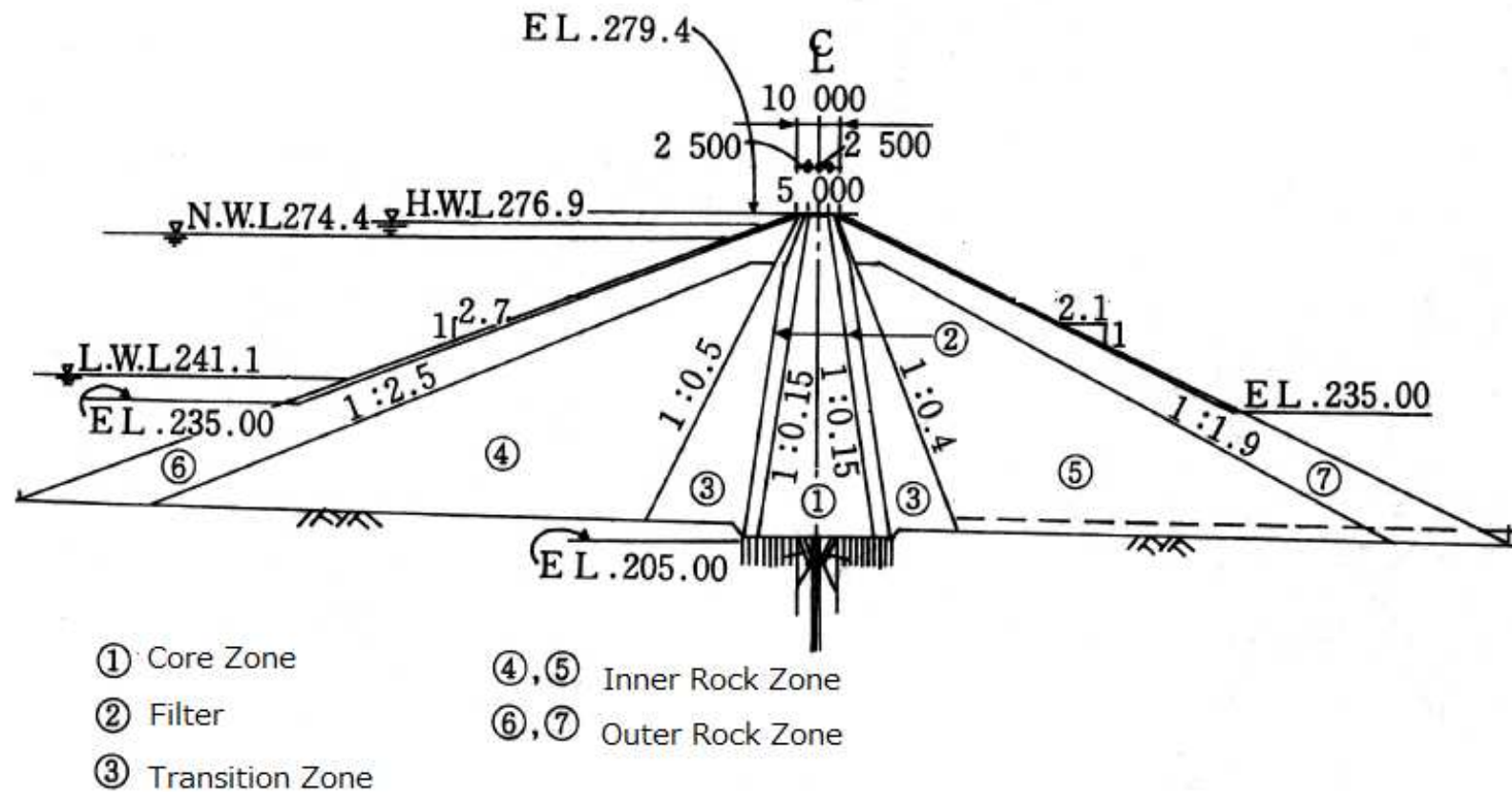
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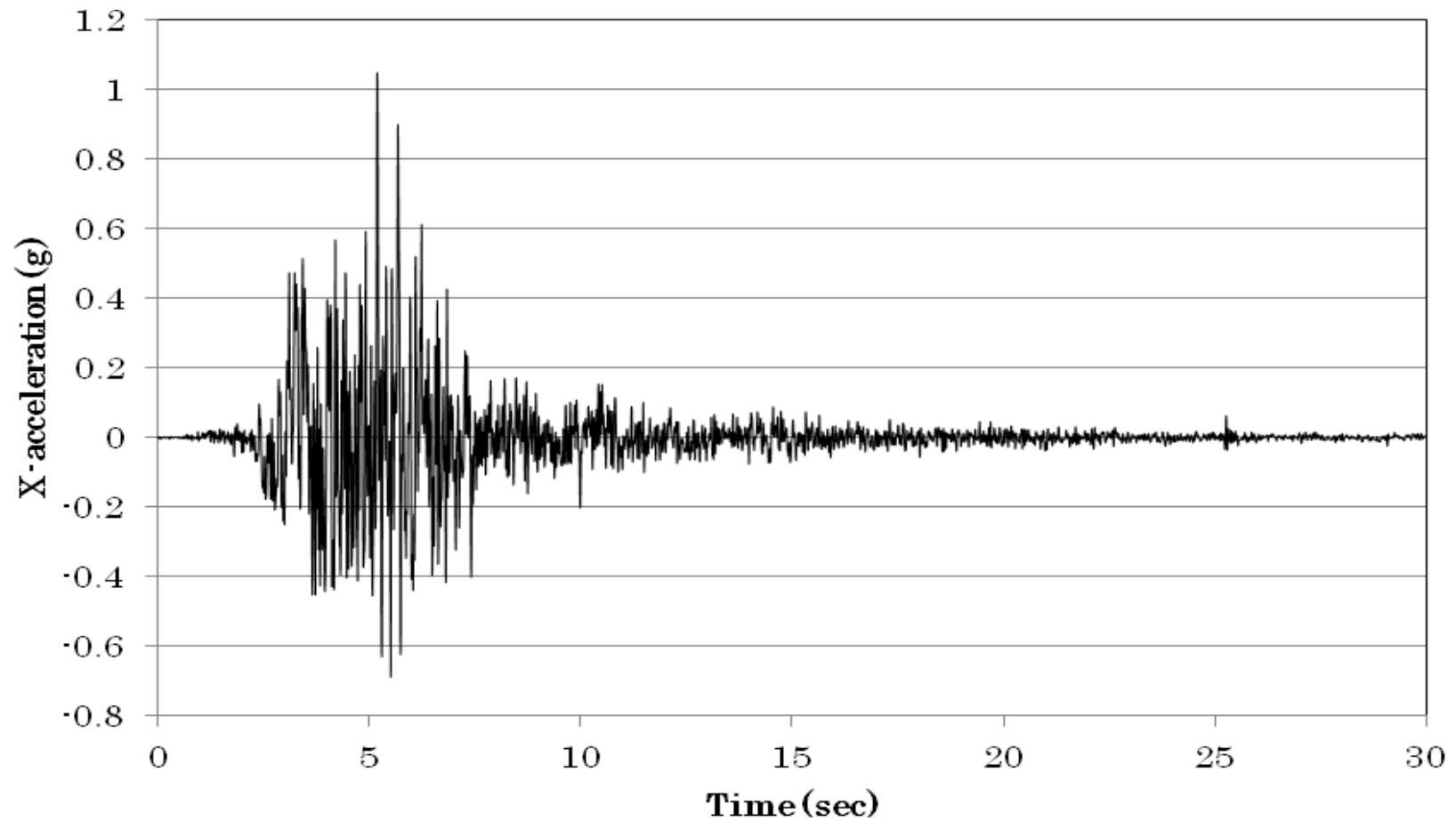
*4.2 Oogaki dam dynamic*

## 5 SUMMARY

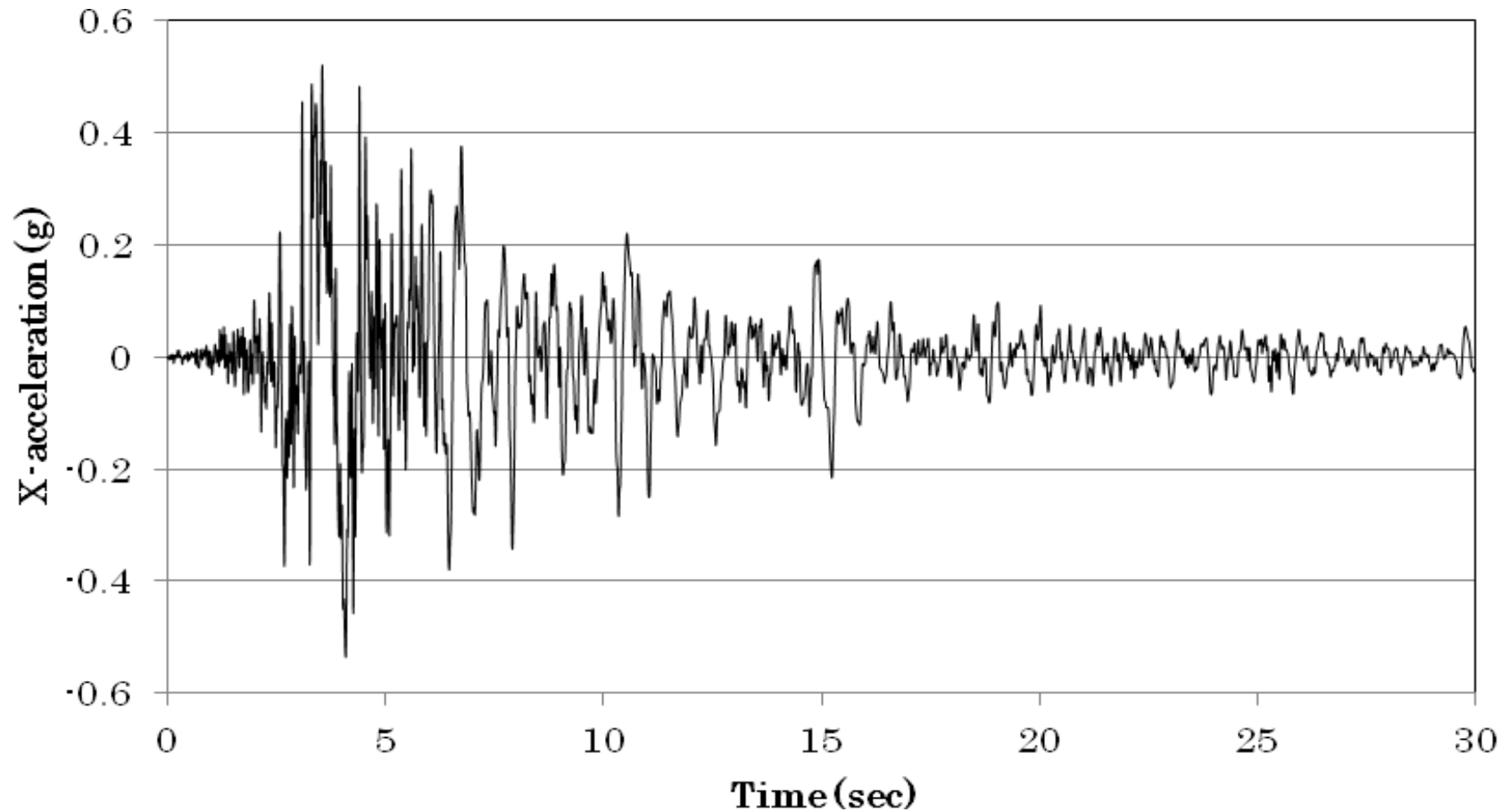
# CROSS SECTION OF ARATOZAWA DAM



# RECORDED ACCELERATION AT THE BASE OF ARATOZAWA DAM

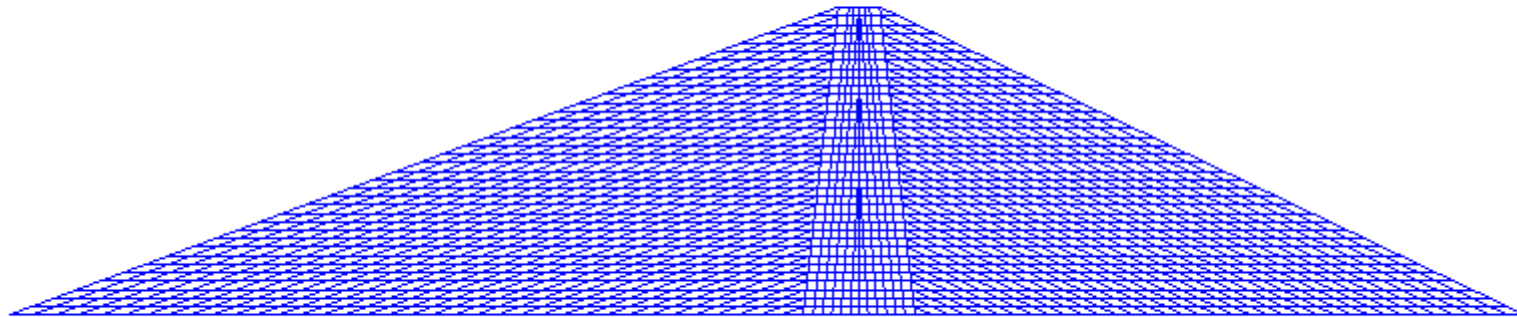


# RECORDED ACCELERATION AT THE CREST OF ARATOZAWA DAM

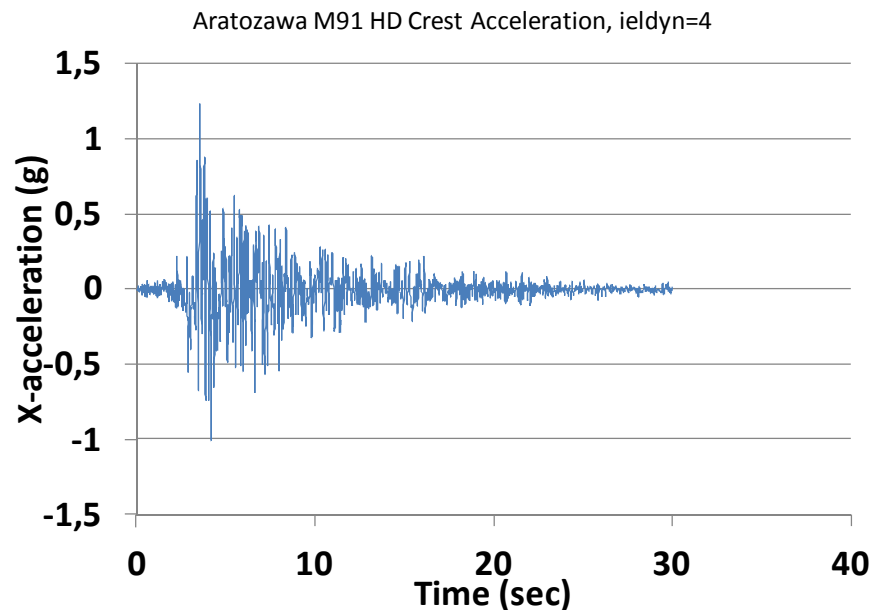




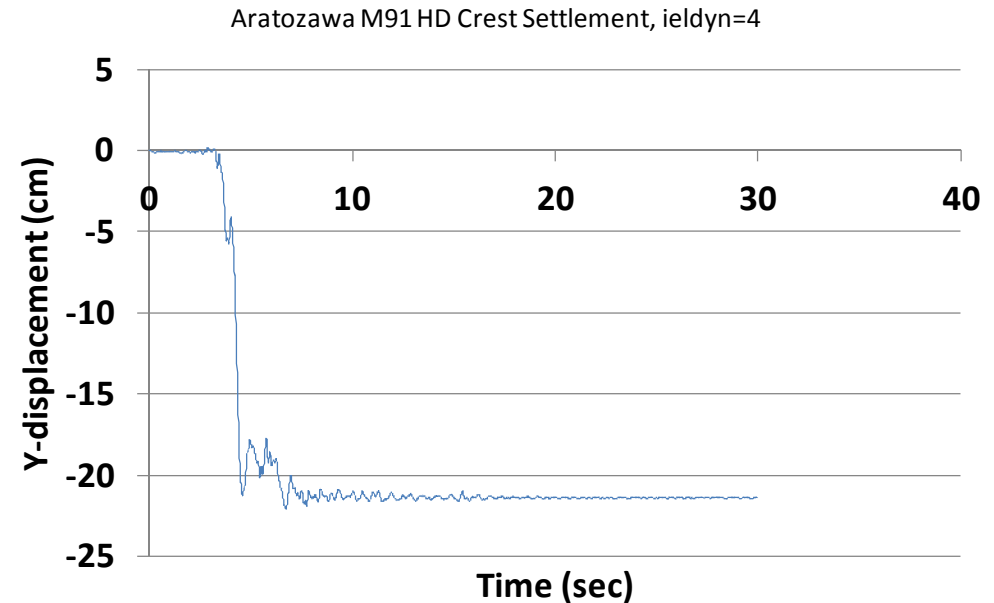
# ARATOZAWA DAM FINITE ELEMENT MODEL



# COMPUTED HORIZONTAL ACCELERATION AND SETTLEMENT AT CREST OF ARATOWAWA DAM TWO DIMENSIONAL ANALYSIS



Computed horizontal acceleration at the crest of dam

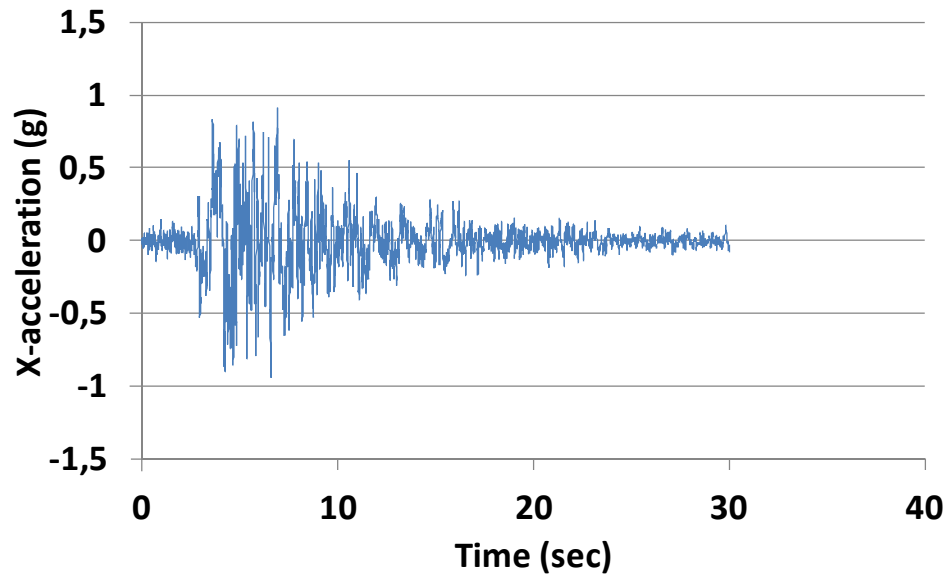


Computed settlement at the crest of dam

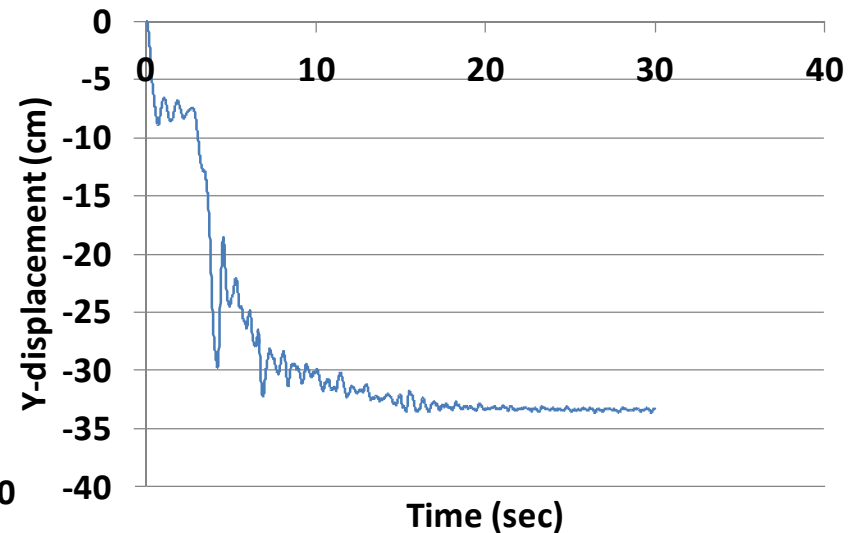
Simple strain softening constitutive model ( peak strength, residual strength, shear band thickness & softening rate are needed)

Elastic limit for shear modulus and damping ratio : reference shear strain for Hardin-Drnevich equation

# COMPUTED HORIZONTAL ACCELERATION AND SETTLEMENT AT CREST OF ARATOZAWA DAM TWO DIMENSIONAL ANALYSIS



Computed horizontal acceleration at the crest of dam



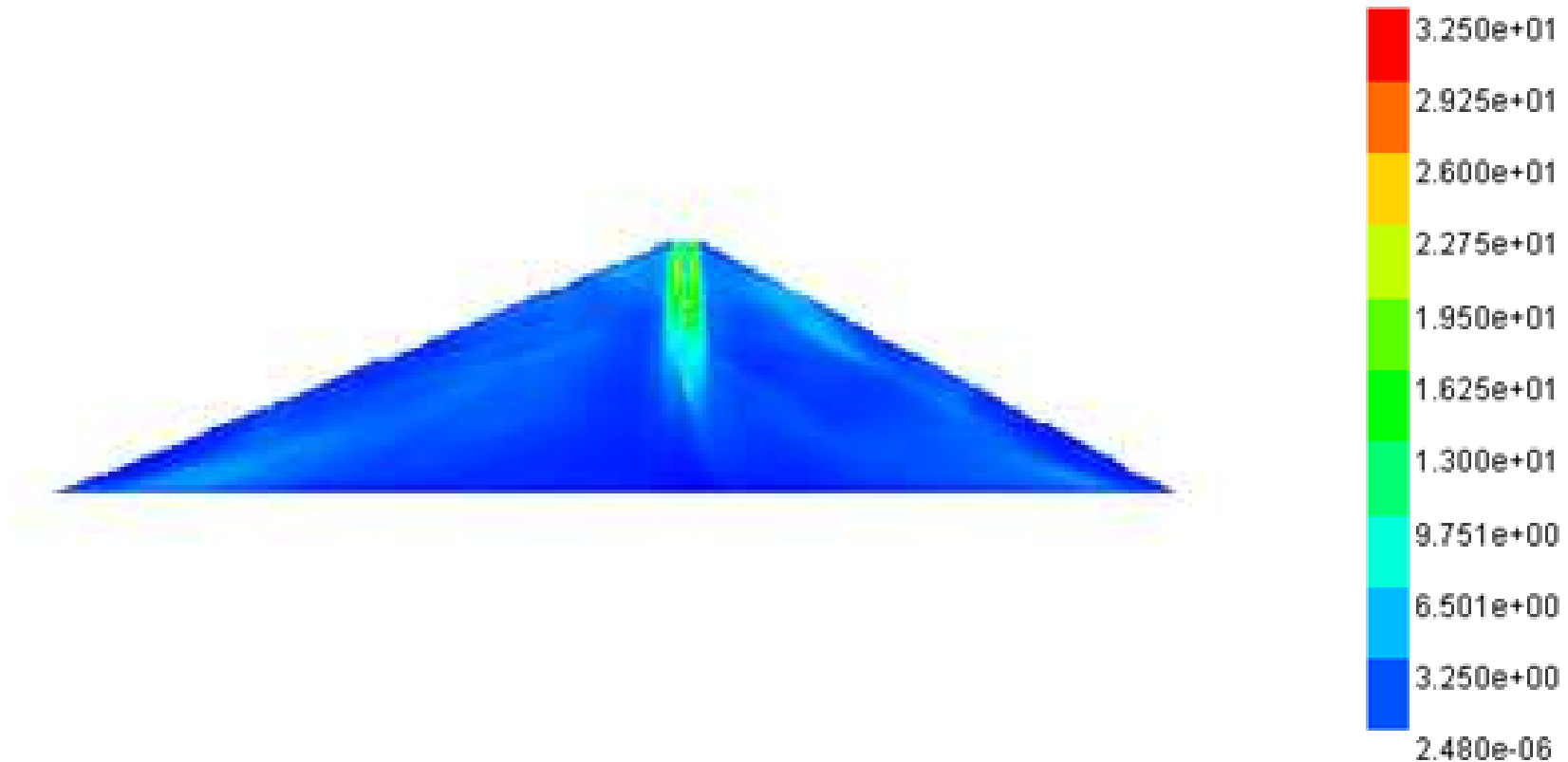
Computed settlement at the crest of dam

## Kinematic hardening constitutive model

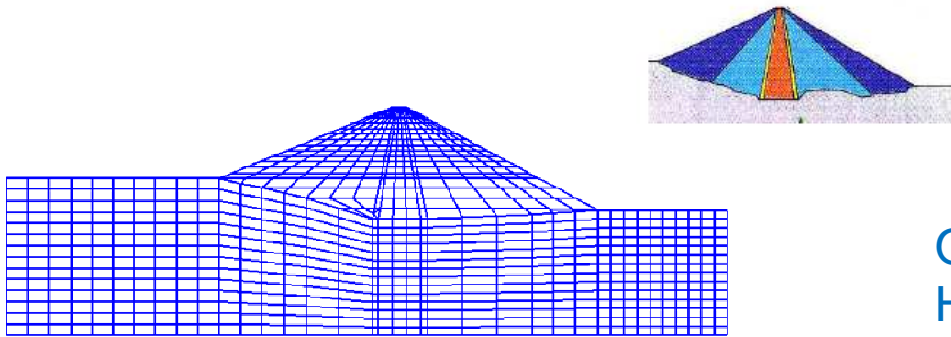
**Effective stress analysis: Core zone is undrained, Rock zones are drained**

Elastic shear modulus and damping ratio : reference shear strain for Hardin-Drnevich equation

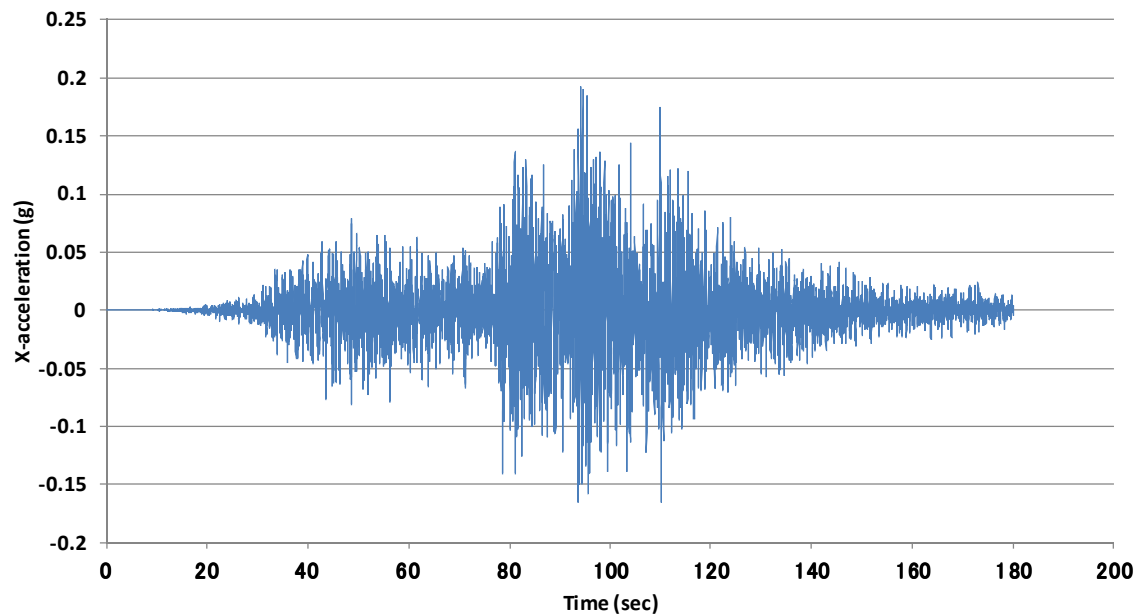
# COMPUTED ARATOZAWA DAM MAXIMUM SHEAR STRAIN DISTRIBUTION (%)



# OOGAKI DAM DYNAMIC ANALYSIS BY SIMPLE ELASTO-PLASTIC MODEL



Oogaki Dam finite element mesh.  
Height of this dam is 85.5 m. Blue  
shows transition zone



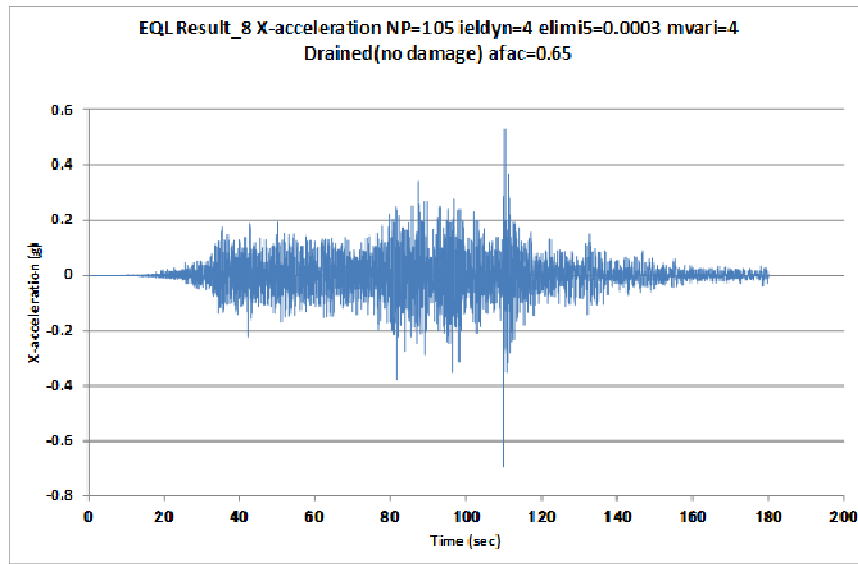
Estimated input acceleration  
at the base of the dam

# COMPUTED RESULTS BY SIMPLE CONSTITUTIVE MODEL TRANSITION ZONE IS DRAINED

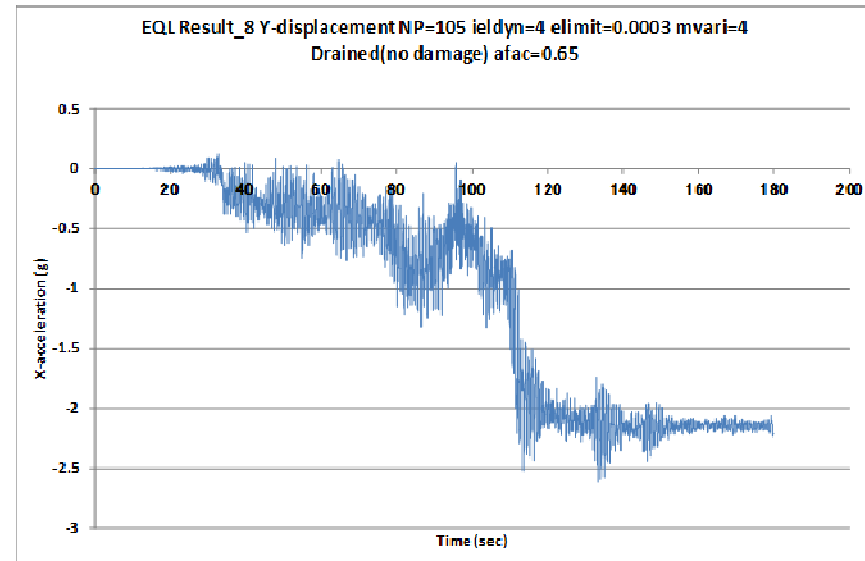
Core zone is undrained and another zones are completely drained

Transition zone material properties:

$\phi_p = 42.1^\circ$ ,  $\phi_r = 34.0^\circ$ ,  $B = 0.7$ ,  $C = 0.6$ ,  $D = 0.7$ , cohesion = 76 kPa, shear band thickness = 4 cm,  $G_E = 1200.0$ , Rayleigh damping alpha = 0, elastic limit = 0.0003.



Computed crest acceleration



Computed crest settlement

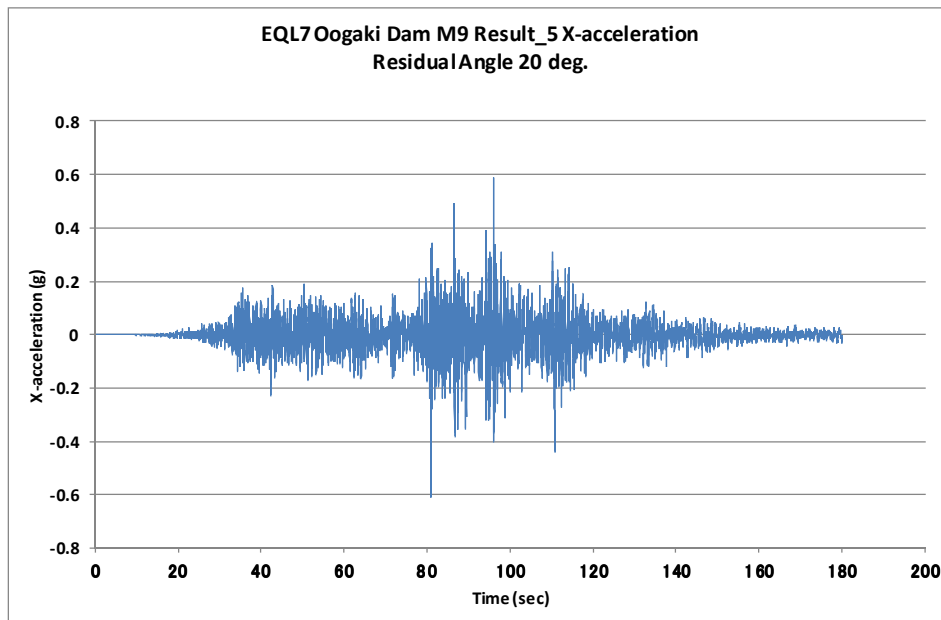
# COMPUTED RESULTS BY SIMPLE CONSTITUTIVE MODEL

## TRANSITION ZONE IS UNDRAINED

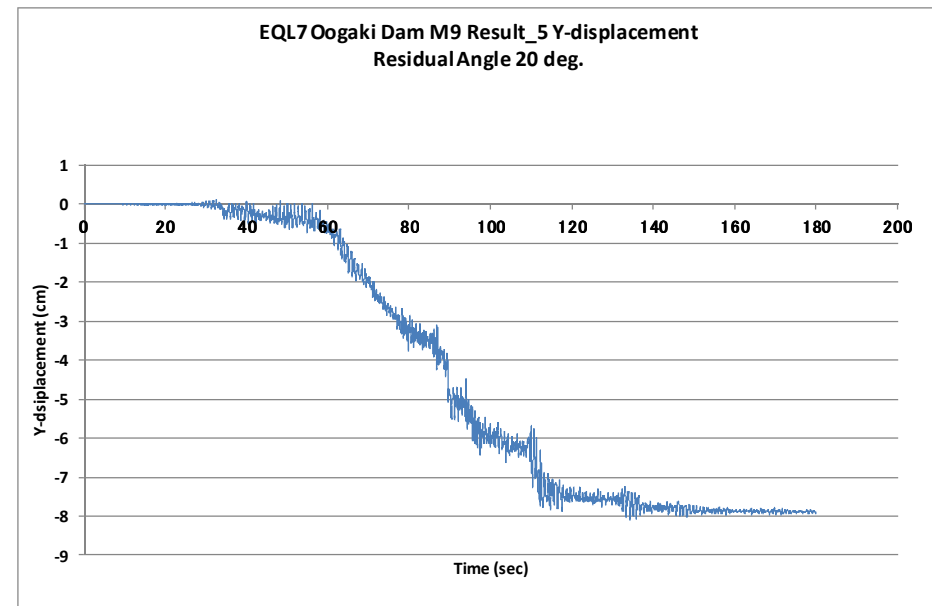
Core zone is undrained and another zones are completely drained

Transition zone material properties:

$\phi_p = 36.6^\circ$ ,  $\phi_r = 20.0^\circ$ ,  $B = 0.7$ ,  $C = 0.6$ ,  $D = 0.7$ , cohesion = 353 kPa, shear band thickness = 4 cm,  $G_E = 1200.0$ , Rayleigh damping alpha = 0, elastic limit = 0.0003.



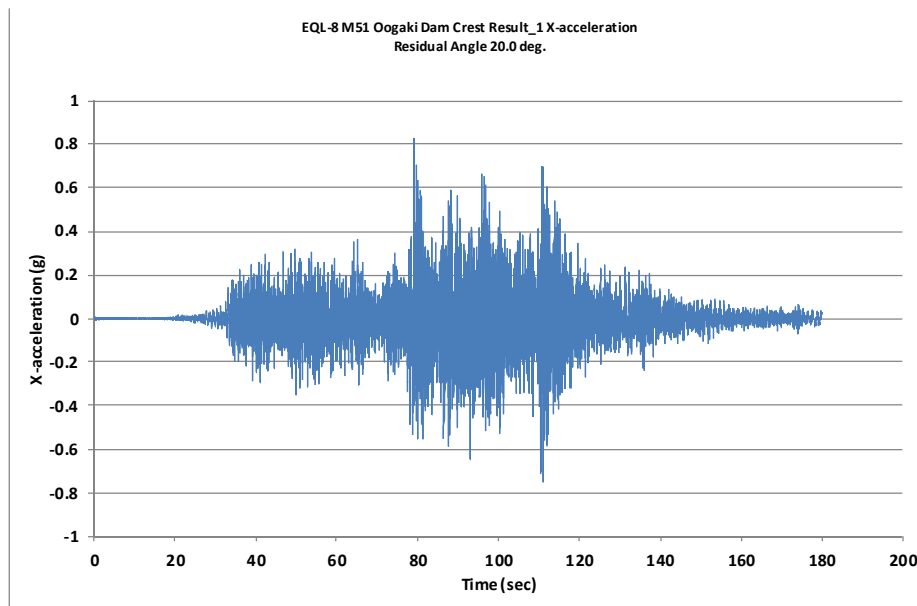
Computed crest acceleration



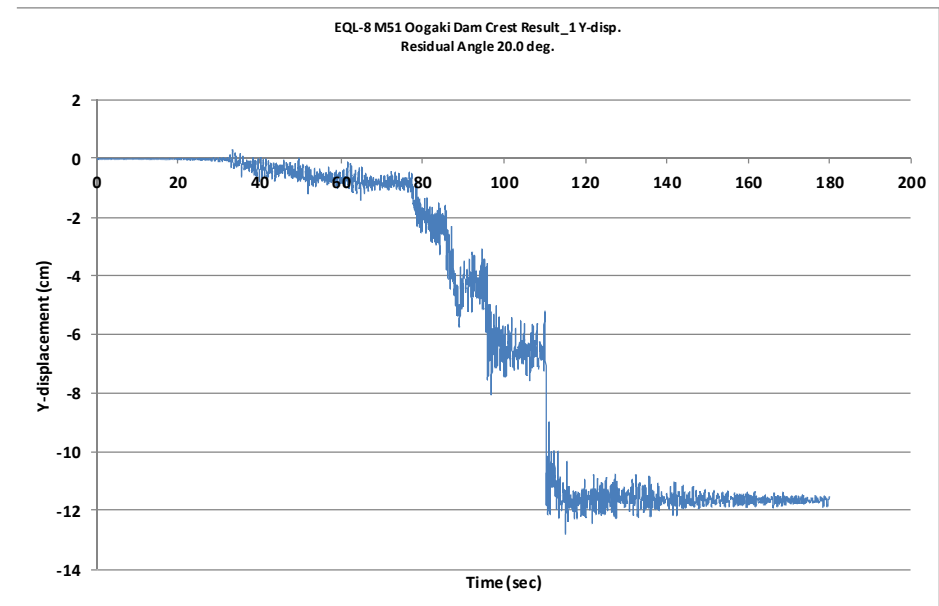
Computed crest settlement

# DYNAMIC RESPONSE ANALYSIS BY KINEMATIC HARDENING ELASTO-PLASTIC MODEL (TOTAL STRESS ANALYSIS)

$c = 374 \text{ kPa}$ ,  $\phi_p = 36.6^\circ$ ,  $\phi_r = 20.0^\circ$ ,  $\varepsilon_f = 0.03$ ,  $\varepsilon_r = 0.6$ ,  $a_f = 5.0$ ,  
 $m = 1$ ,  $l = 0.5$ ,  $n = 1.0$ , Thickness of shear zone = 4 cm. The factor of plastic  
parameter is 3000.0.



Computed crest acceleration by total stress analysis

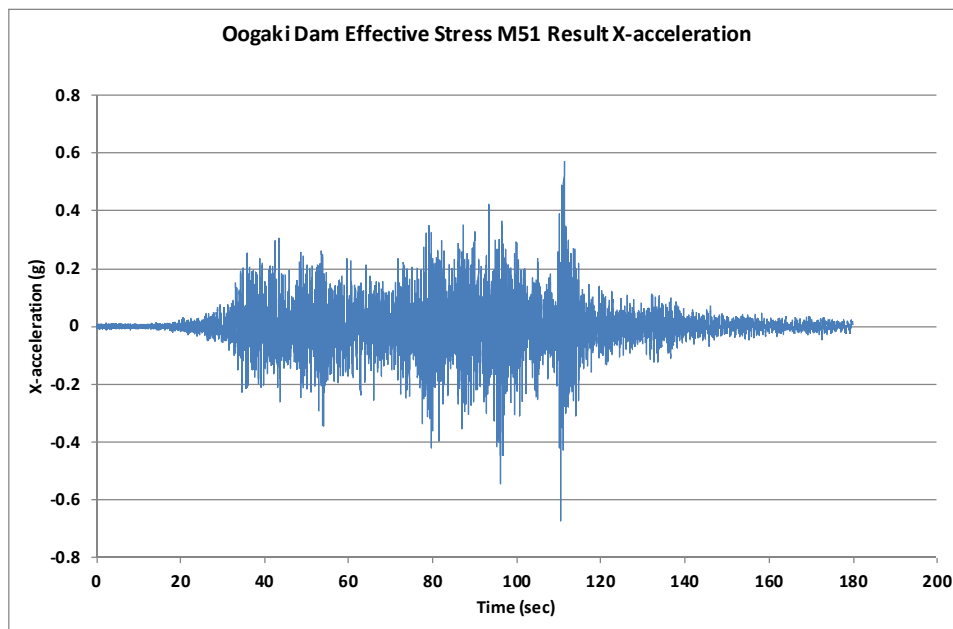


Computed crest settlement by total stress analysis

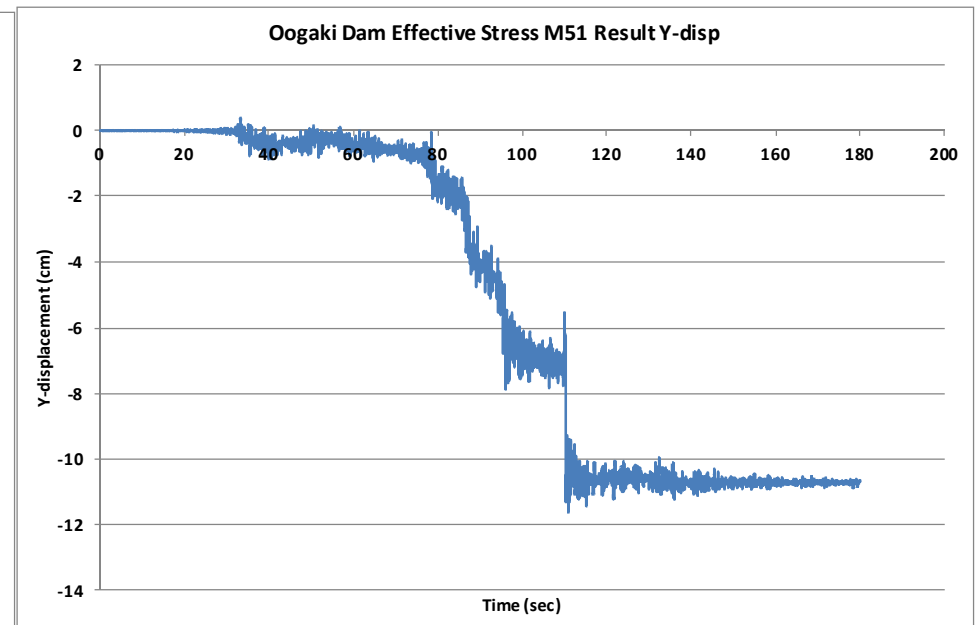


# DYNAMIC RESPONSE ANALYSIS BY KINEMATIC HARDENING ELASTO-PLASTIC MODEL (EFFECTIVE STRESS ANALYSIS)

$c = 11 \text{ kPa}$ ,  $\phi_p = 42.0^\circ$ ,  $\phi_r = 34.0^\circ$ ,  $\varepsilon_f = 0.03$ ,  $\varepsilon_r = 0.6$ ,  $a_f = 5.0$ ,  
 $m = 0.5$ ,  $n = 1.0$ , Thickness of shear zone = 4 cm.

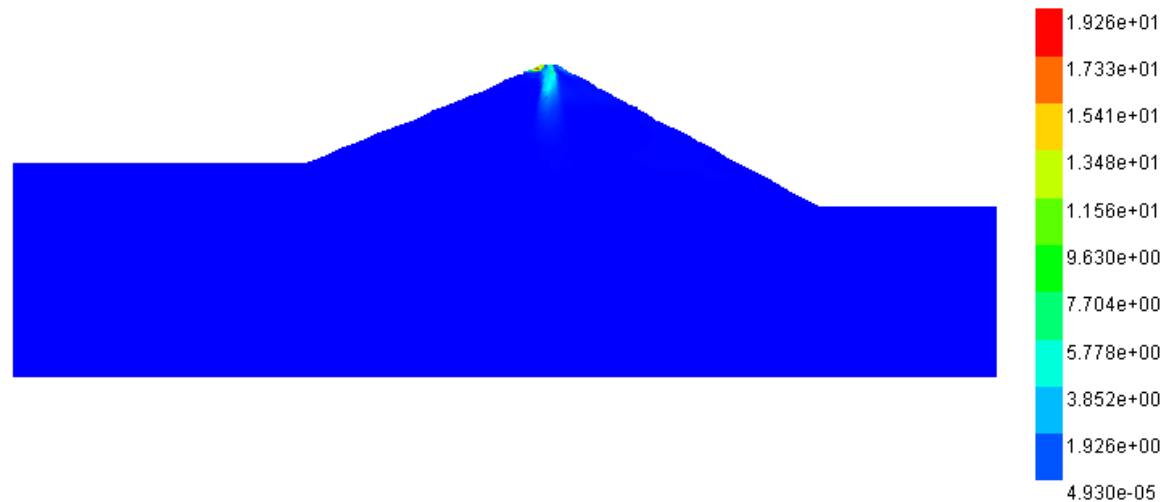


Computed crest acceleration by  
effective stress analysis



Computed crest settlement by  
effective stress analysis

# COMPUTED MAXIMUM SHEAR STRAIN (%) BY KINEMATIC HARDENING MODEL EFFECTIVE STRESS ANALYSIS



# SUMMARY

A dynamic progressive failure analysis of a small embankment dam using dry sand on shaking table is carried out. The acceleration simulating El Centro earthquake is applied to the base of shaking table. The computed acceleration at the crest of model dam is compared to the observed one and the computed displacement is also verified by the observed displacement.

A shear banding constitutive model incorporating a characteristics length of shear band is necessary. Both a simple strain softening constitutive model and a kinematic hardening model are also applicable to total stress dynamic response analyses by applying incompressible condition in case of saturated soils.

The computation of real fill-type dam is also carried out by total stress elasto-plastic constitutive model and effective stress constitutive model by taking into account the pore water build-up.

These computations are carried out by computer code **NONSOLAN**: Nonlinear Solid and Soil Analysis

**THANK YOU FOR YOUR ATTENSION**

# DERIVATION OF STRESS-STRAIN RELATION

Total strain increment

$$d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}^e + s\lambda d\boldsymbol{\varepsilon}^p$$

$$d\boldsymbol{\varepsilon} = \mathbf{D}^{e-1} d\boldsymbol{\sigma} + s\lambda \frac{\partial \Phi}{\partial \boldsymbol{\sigma}}$$

Multiply  $\mathbf{D}^e$

$$\mathbf{D}^e d\boldsymbol{\varepsilon} = d\boldsymbol{\sigma} + s\lambda \mathbf{D}^e \mathbf{b}$$

$$\text{where} \quad \left( \mathbf{b} = \frac{\partial \Phi}{\partial \boldsymbol{\sigma}} \right)$$

$$d\boldsymbol{\sigma} = \mathbf{D}^e d\boldsymbol{\varepsilon} - s\lambda \mathbf{D}^e \mathbf{b}$$

$$s = F_b / F_e \quad F_e \text{ is the area of the element}$$

$$F_b \text{ is the area of a single shear band in each element}$$

# PARAMETER ( $\lambda$ )

$$d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}^e + s\lambda d\boldsymbol{\varepsilon}^p$$

Stress and strain must satisfy the yield condition

$$d\boldsymbol{\varepsilon} = \mathbf{D}^{e-1} d\boldsymbol{\sigma} + s\lambda \frac{\partial \Phi}{\partial \boldsymbol{\sigma}}$$

Multiply both sides

$$\mathbf{a}^T \mathbf{D}^e$$

$$\begin{aligned} \mathbf{a}^T \mathbf{D}^e d\boldsymbol{\varepsilon} &= \mathbf{a}^T d\boldsymbol{\sigma} + s\lambda \mathbf{a}^T \mathbf{D}^e \mathbf{b} \\ &= (\mathbf{A} + s\mathbf{a}^T \mathbf{D}^e \mathbf{b}) \lambda \end{aligned}$$

$$\lambda = \frac{\mathbf{a}^T \mathbf{D}^e d\boldsymbol{\varepsilon}}{\mathbf{A} + s\mathbf{a}^T \mathbf{D}^e \mathbf{b}}$$

$$f(\boldsymbol{\sigma}, \kappa) = 0$$

$$df = 0 \quad (\text{consistency condition})$$

$$\frac{\partial f}{\partial \boldsymbol{\sigma}} d\boldsymbol{\sigma} + \frac{\partial f}{\partial \kappa} d\kappa = 0$$

$$\mathbf{a}^T d\boldsymbol{\sigma} - \mathbf{A} \lambda = 0$$

$$d\boldsymbol{\sigma} = \mathbf{a}^{T-1} \mathbf{A} \lambda$$

$$\left( \mathbf{a}^T = \frac{\partial f}{\partial \boldsymbol{\sigma}}, \quad \mathbf{A} = -\frac{1}{\lambda} \frac{\partial f}{\partial \kappa} d\kappa \right)$$

# STRESS-STRAIN RELATION

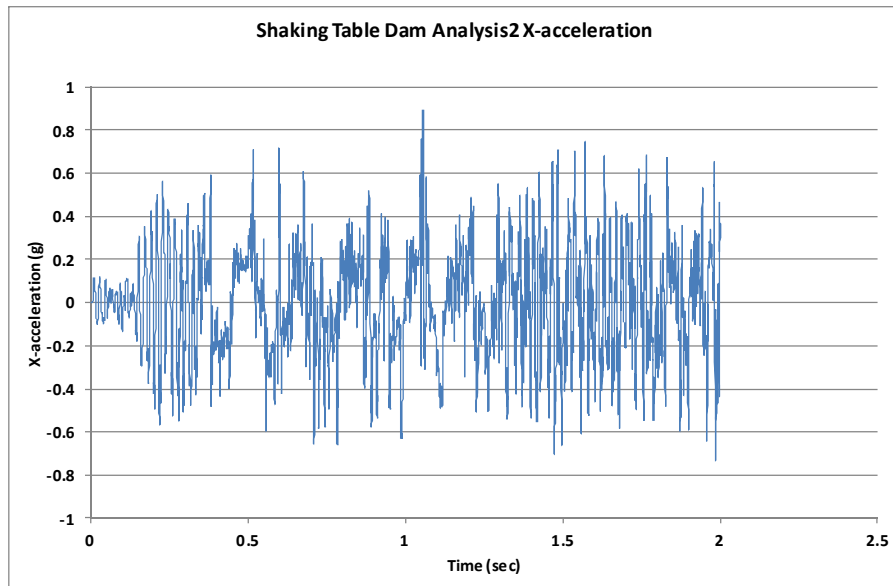
$$d\boldsymbol{\sigma} = \mathbf{D}^e d\boldsymbol{\varepsilon} - s\lambda \mathbf{D}^e \mathbf{b} \quad \text{Substituting } \lambda \quad \left( \lambda = \frac{\mathbf{a}^T \mathbf{D}^e d\boldsymbol{\varepsilon}}{\mathbf{A} + s\mathbf{a}^T \mathbf{D}^e \mathbf{b}} \right)$$

We can obtain the next stress-strain relation

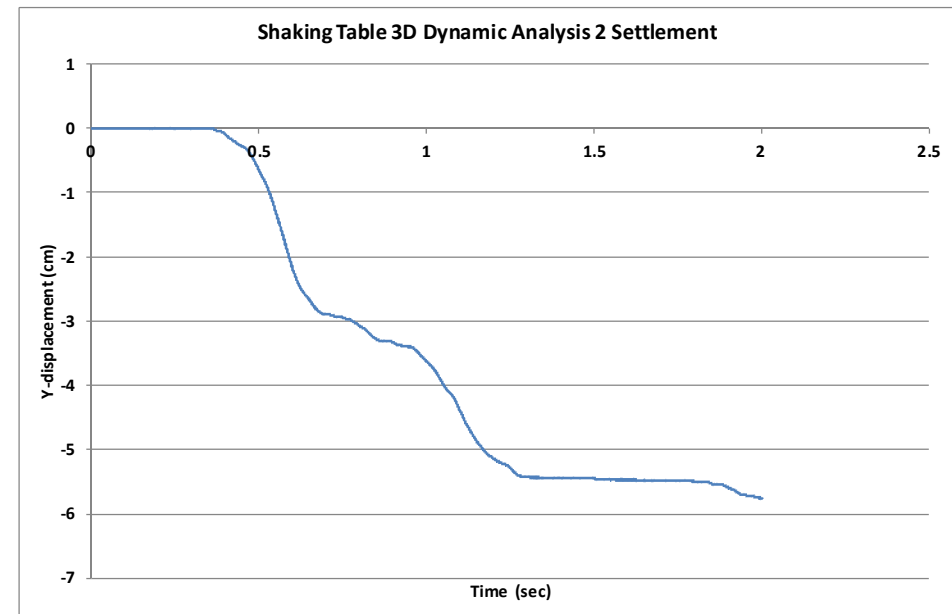
$$d\boldsymbol{\sigma} = \left( \mathbf{D}^e - \frac{s\mathbf{D}^e \mathbf{b} \mathbf{a}^T \mathbf{D}^e d\boldsymbol{\varepsilon}}{\mathbf{A} + s\mathbf{a}^T \mathbf{D}^e \mathbf{b}} \right) d\boldsymbol{\varepsilon}$$

$$\left( \mathbf{a}^T = \frac{\partial f}{\partial \boldsymbol{\sigma}} \quad , \quad \mathbf{b} = \frac{\partial \Phi}{\partial \boldsymbol{\sigma}} \quad , \quad \mathbf{A} = -\frac{1}{\lambda} \frac{\partial f}{\partial \boldsymbol{\kappa}} d\boldsymbol{\kappa} \right)$$

# COMPUTED HORIZONTAL ACCELERATION AND SETTLEMENT AT CREST OF MODEL DAM THREE DIMENSIONAL ANALYSIS



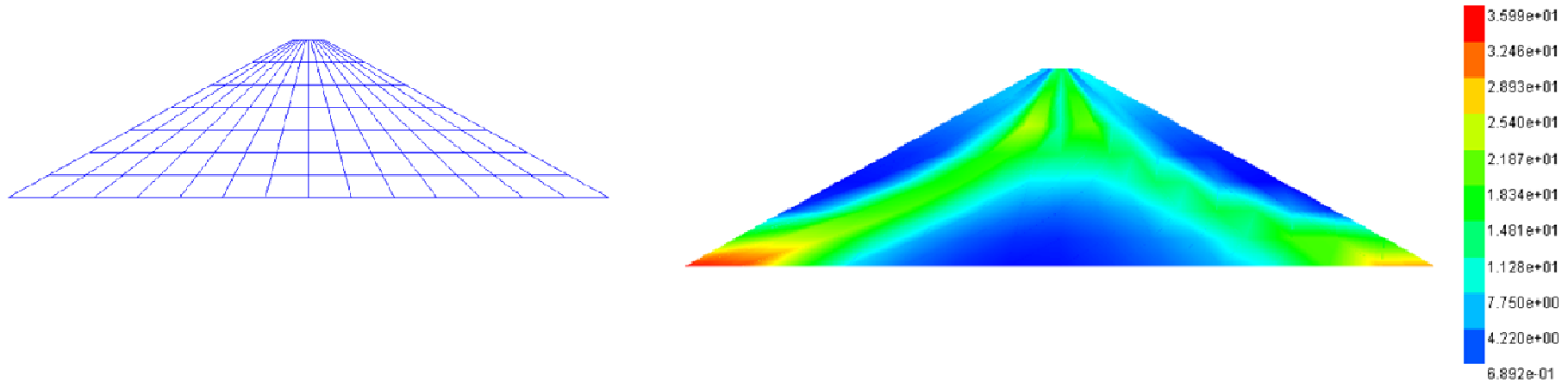
Computed horizontal acceleration at the crest of model dam



Computed settlement at the crest of model dam



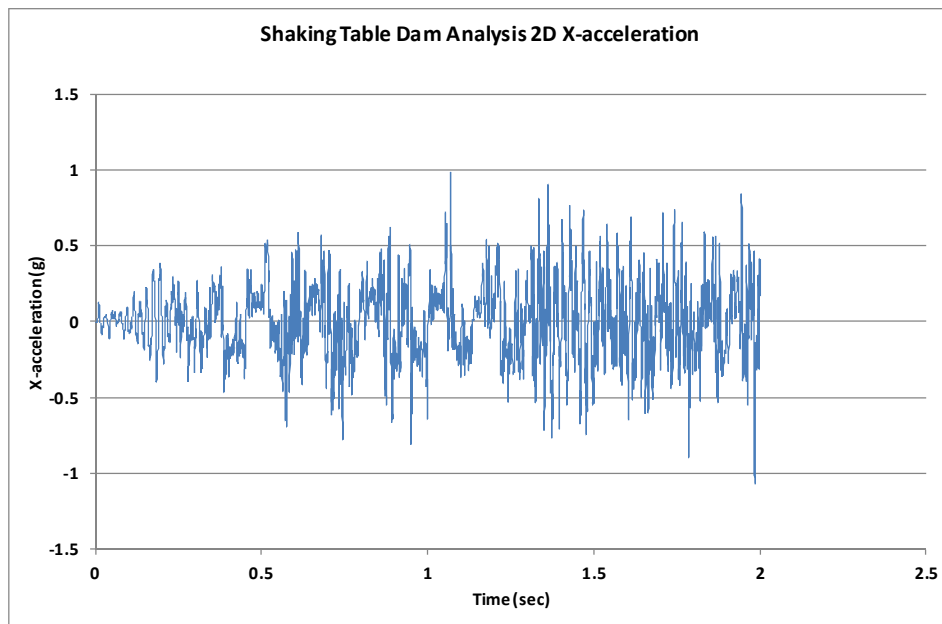
# TWO DIMENSIONAL FINITE ELEMENT ANALYSIS OF MODEL DAM ON SHAKING TABLE TEST



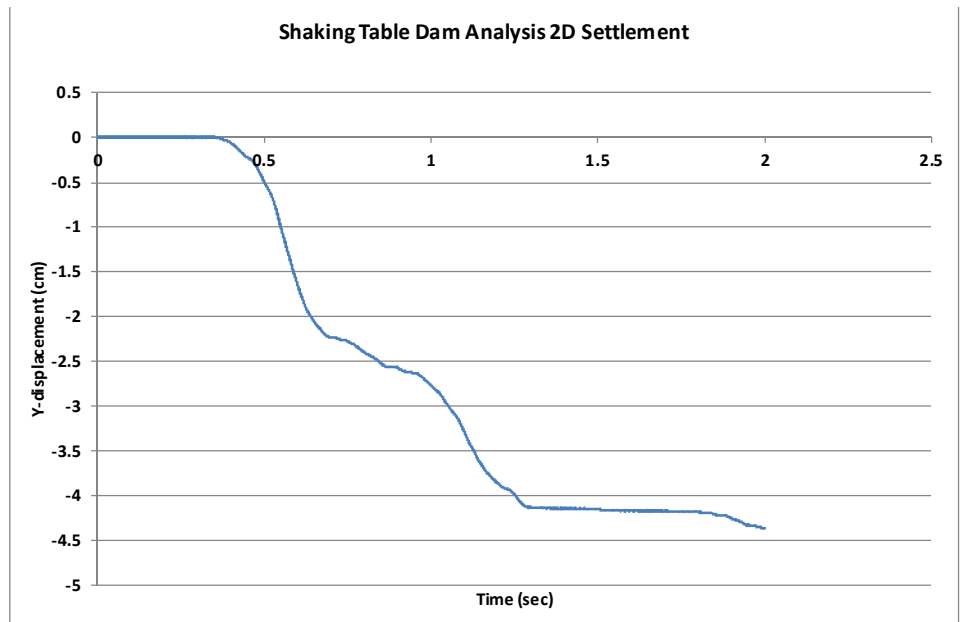
Two dimensional finite element mesh

Computed maximum shear strain of model dam  
after shaking (peak strain 30%)

# COMPUTED HORIZONTAL ACCELERATION AND SETTKEMENT AT CREST OF MODEL DAM TWO DIMENSIONAL ANALYSIS

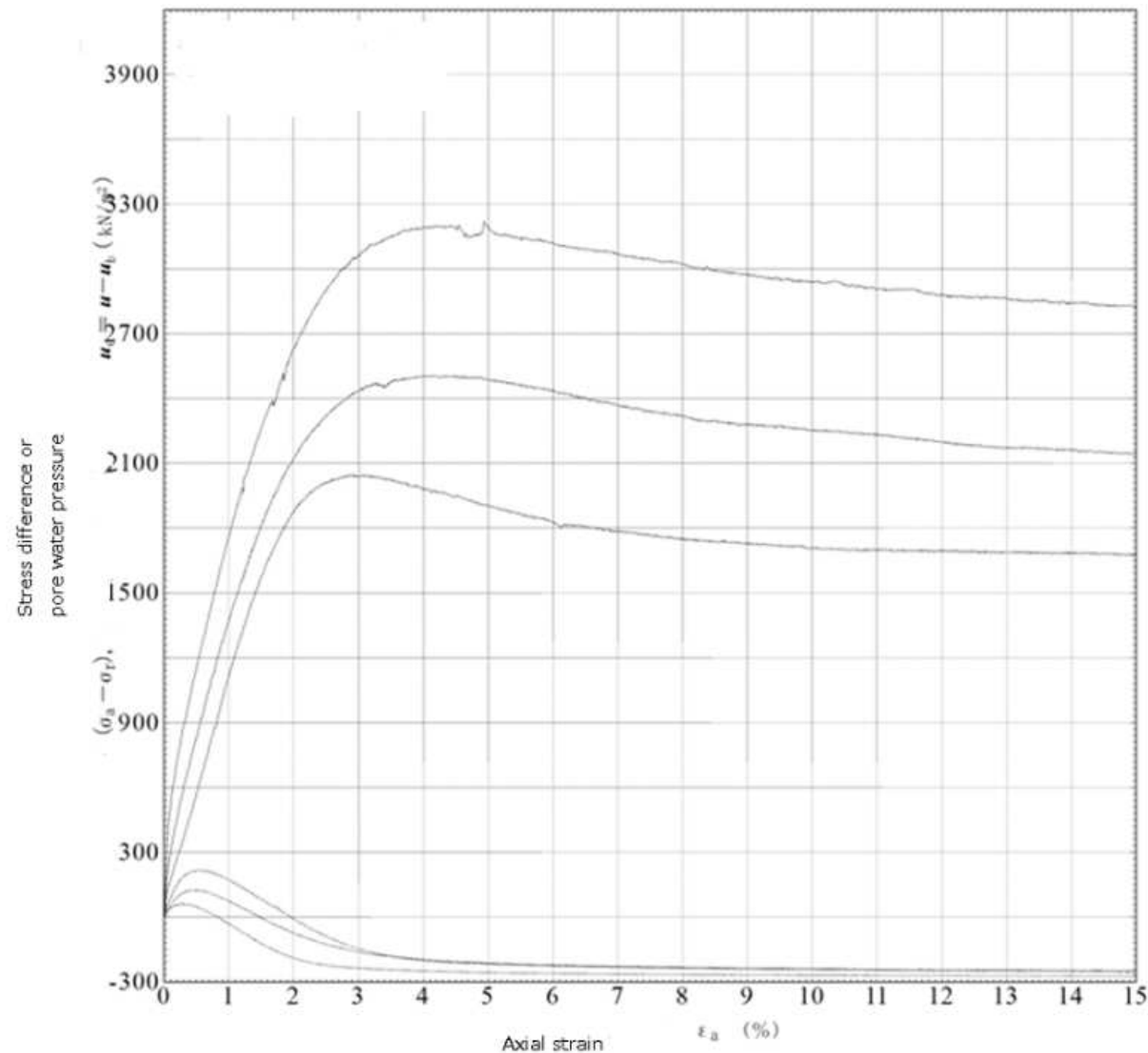


Computed horizontal acceleration at the crest of model dam



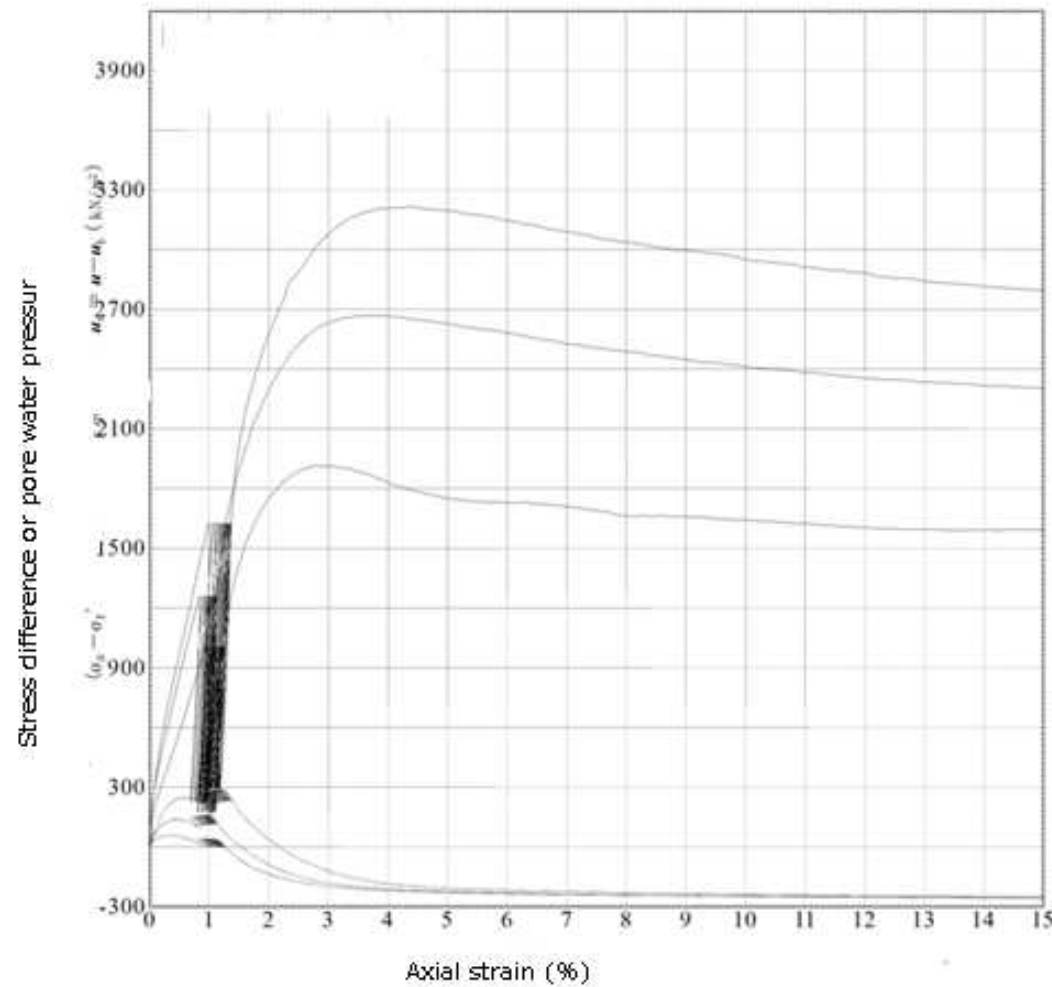
Computed settlement at the crest of model dam

# CONSOLIDATED UNDRAINED STRESS-STRAIN RELATION OOGAKI DAM ROCK MATERIAL (TRANSITION ZONE)



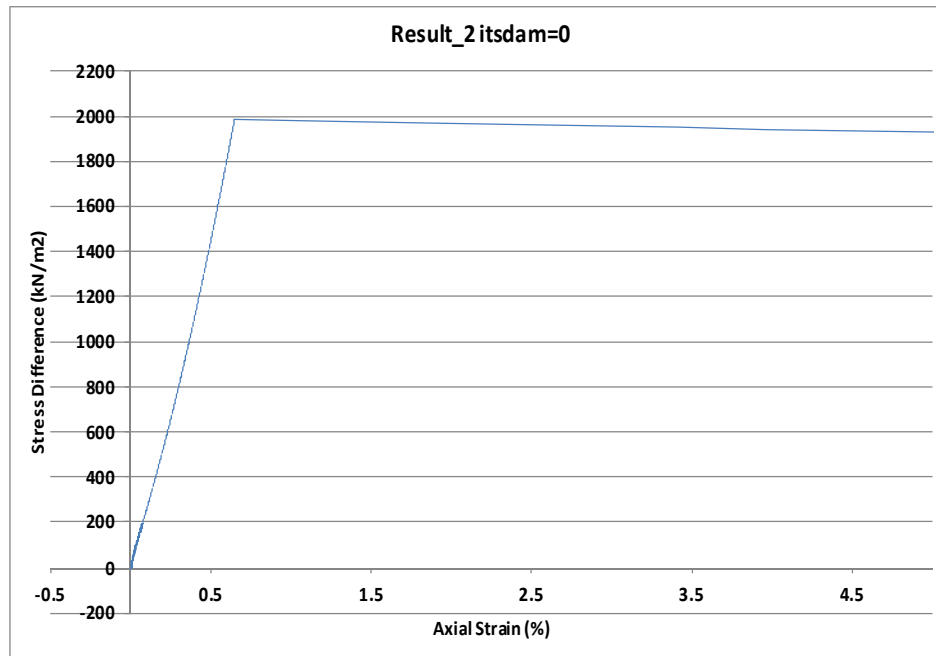
Confining pressure: 200,400,600( kN/m<sup>2</sup>) monotonic loading

# CONSOLIDATED UNDRAINED STRESS-STRAIN RELATION OOGAKI DAM ROCK MATERIAL (TRANSITION ZONE)

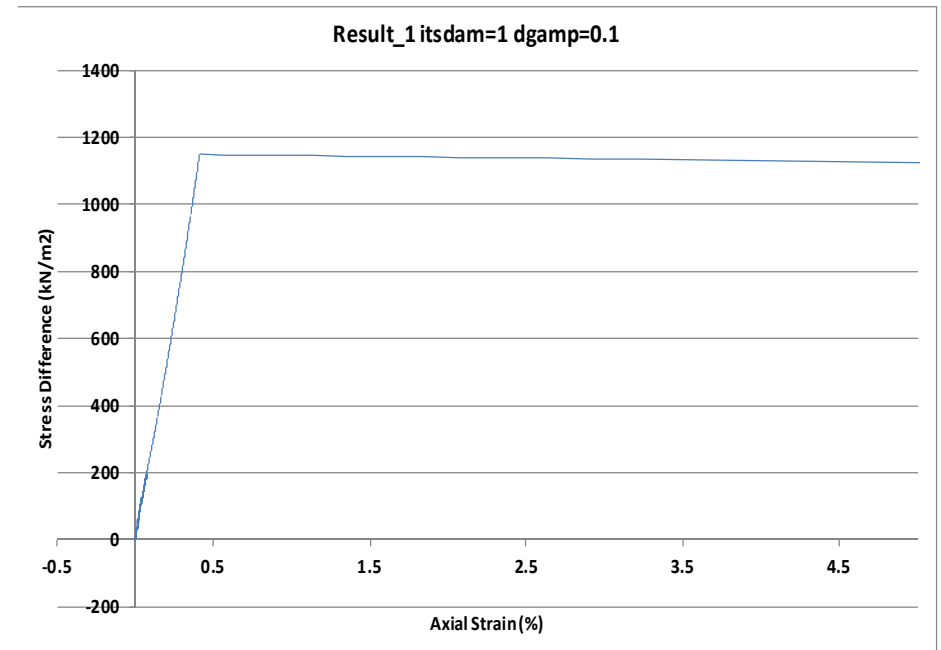


Confining pressure: 200,400,600( kN/m<sup>2</sup>)    Cyclic and monotonic loading

# COMPUTED STRESS-STRAIN RELATION BY SIMPLE CONSTITUTIVE MODEL



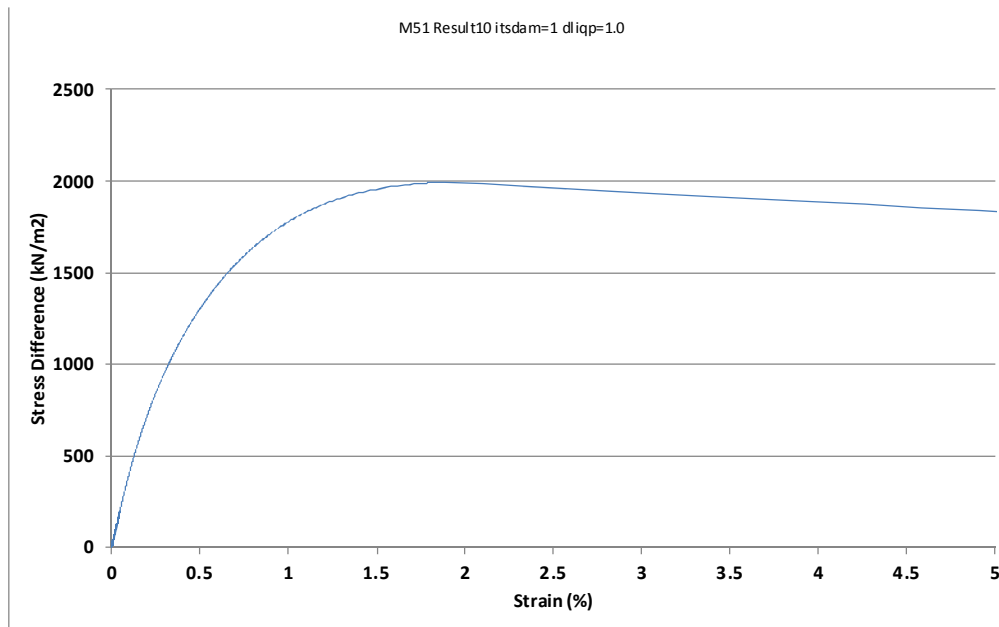
a)



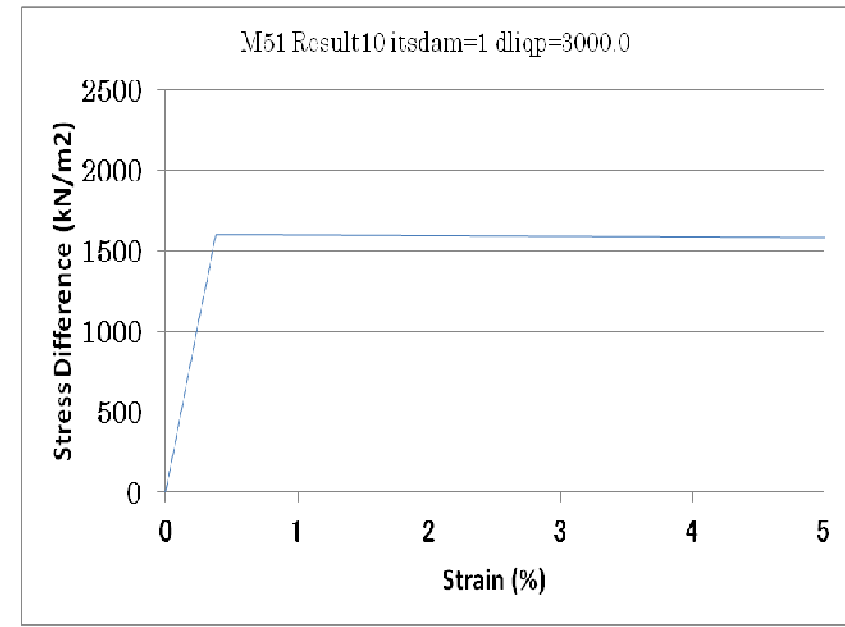
b)

Confining pressure is 200 (kN/m<sup>2</sup>) (applied cyclic load: 0 - 200kN/m<sup>2</sup>)  $\phi_p = 36.6^\circ$ ,  $\phi_r = 35.0^\circ$ ,  
 $B = 0.7$ ,  $C = 0.6$ ,  $D = 0.7$ , cohesion = 354 kPa,  
 a) without factor of equivalent plastic parameter , b) factor is 0.1

# COMPUTED STRESS-STRAIN RELATION BY KINEMATIC HARDENING ELASTO-PLASTIC MODEL



a)



b)

$\phi_p = 36.6^\circ$  ,  $\phi_r = 20.0^\circ$  ,  $\varepsilon_f = 0.03$  ,  $\varepsilon_r = 0.6$  ,  $a_f = 5.0$  ,  $m = 1$  ,  $l = 0.5$  ,  $n = 1.0$  ,  
 $c = 374 \text{ kPa}$  , factor for plastic parameter  $\kappa'$  : a) = 1.0 , b) = 3000.0

In case b) stress-strain behavior is just similar to simple elasto-plastic strain softening model