



**NTNU – Trondheim**  
Norwegian University of  
Science and Technology

# **A researcher's perspective on modelling unbounded domains for earthquake analysis of dams**

**Arnkjell Løkke**

*Norwegian Univ. of Science and Technology (NTNU)*  
*University of California, Berkeley*

# Introduction

---

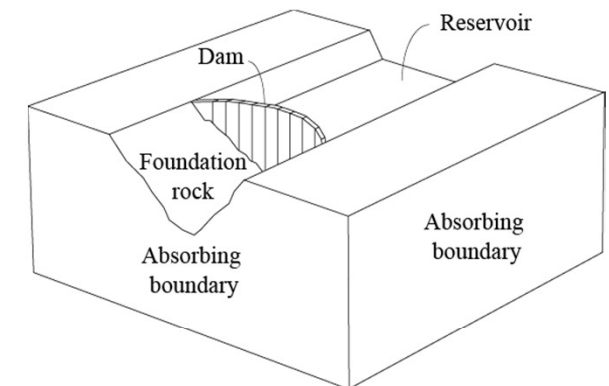
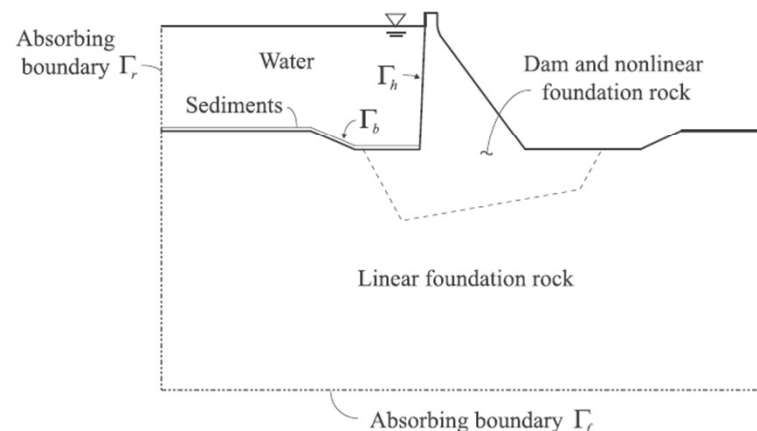
# Focus of my research

## Collaboration between UCB and NTNU

- Supervisor: Professor Anil K. Chopra, UC Berkeley
- Co-supervisor: Amir Kaynia, NTNU

## Objective to formulate and validate accurate procedures for nonlinear response history analysis of concrete dams

- Start with rigorous (theoretical) derivation, then do approximations where necessary
- Procedure for both 2D (gravity) and 3D (arch) dam systems



# Response history analysis of dams is a complex problem

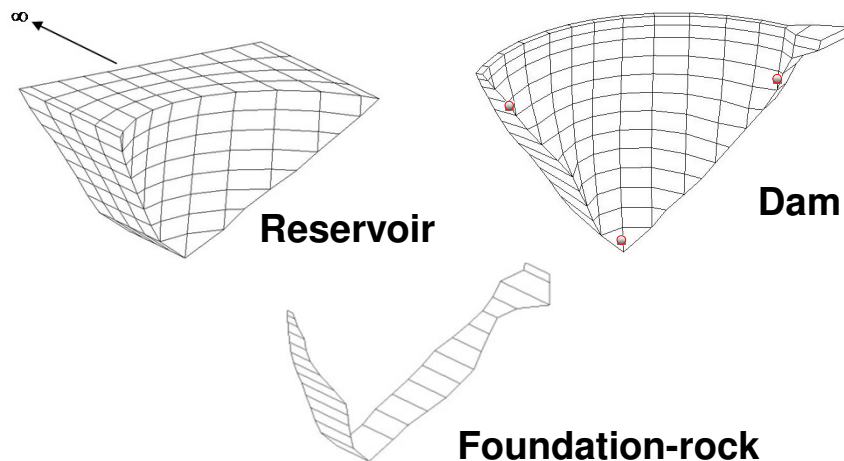


- **Semi-unbounded domains**
  - Foundation-rock and reservoir
- **Interaction effects**
  - Between dam, water, foundation-rock
- **Spatially varying ground motion**
  - How to obtain this motion?
- **Nonlinear mechanisms**
  - Concrete cracking
  - Sliding and separation at joints and interfaces

# Available methods today

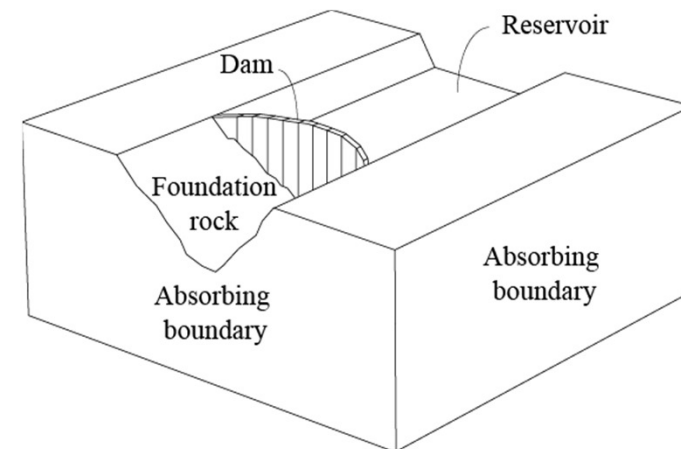
Neither offer a fully satisfactory solution

## Substructure method (frequency domain)



- **Special-purpose software**
  - EAGD84 / EACD-3D
- **Rigorous (analytical) treatment of unbounded domains**
- **Restricted to linear analysis**

## Direct FE method (time domain)

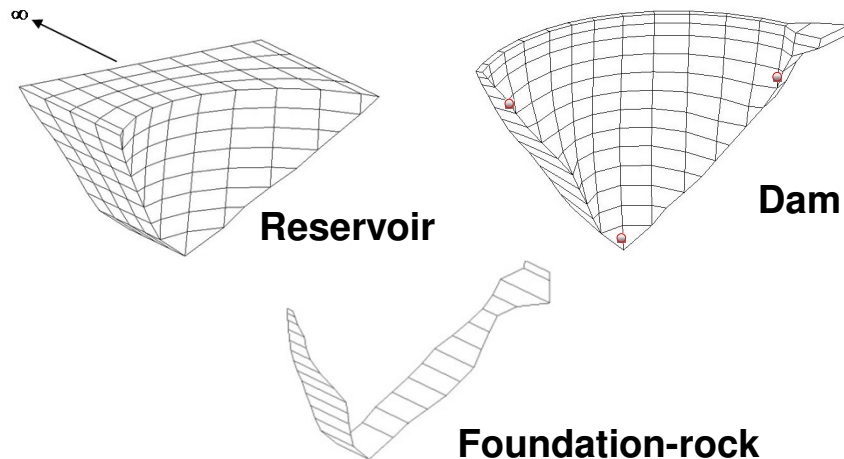


- **Commercial FE software**
  - Abaqus, LS-DYNA, etc.
- **Nonlinear analysis**
- **Often unsatisfactory models for unbounded domains**

# Available methods today

Neither offer a fully satisfactory solution

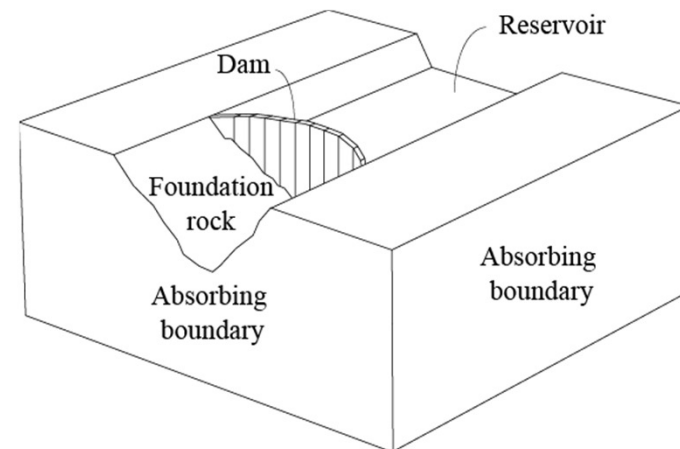
## Substructure method (frequency domain)



- **Special-purpose software**
  - EAGD84 / EACD-3D
- **Rigorous (analytical) treatment of unbounded domains**
- **Restricted to linear analysis**

*Focus of today's presentation*

## Direct FE method (time domain)



- **Commercial FE software**
  - Abaqus, LS-DYNA, etc.
- **Nonlinear analysis**
- **Often unsatisfactory models for unbounded domains**

# Outline of presentation

---

- 1 Theory: Modelling of unbounded domains**
- 2 Results for 2D gravity dam system**
- 3 Summary and recommendations**

**Objective is to highlight pitfalls and provide recommendations for modelling unbounded domains**

# **Theory: Modelling of unbounded domains**

---



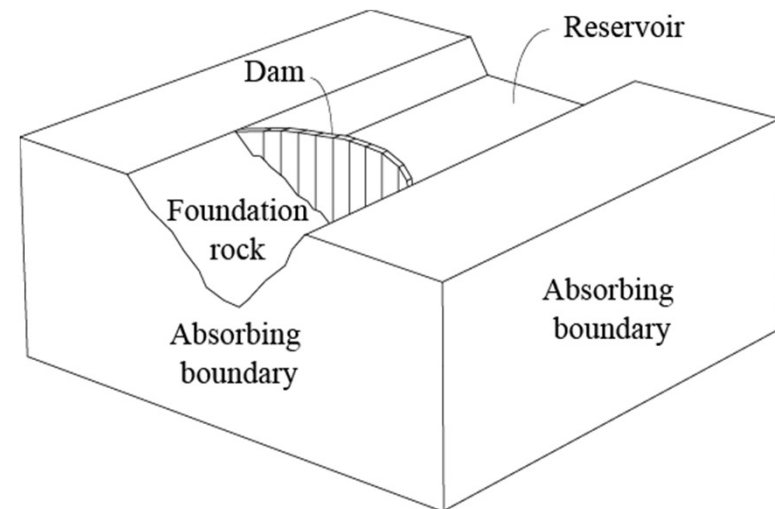
# Unbounded domains requires special attention

Issue 1: Absorbing boundaries must be applied to model

Unbounded domains must be truncated and wave-absorbing boundaries applied at these truncations

Need to satisfy radiation condition at the boundary:

- e.g.  $\frac{\partial u}{\partial x} = \frac{1}{c} \frac{\partial u}{\partial t}$  in the x-direction



# Unbounded domains requires special attention

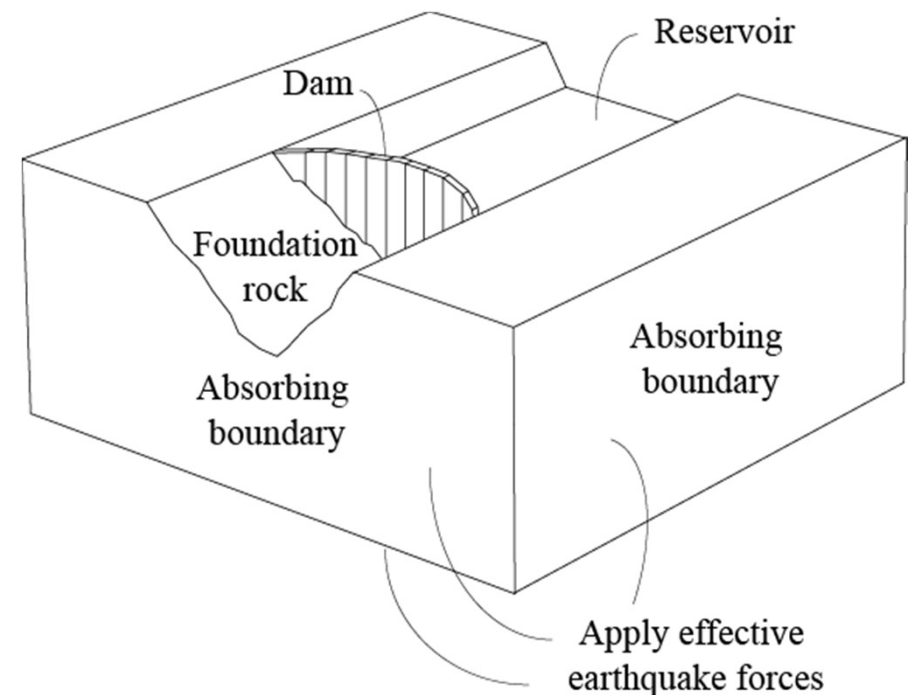
## Issue 2: Seismic input must be specified as Effective Earthquake Forces

**Prescribing displacements at the model truncations will lead to a reflective boundary**

**Instead, Effective Earthquake Forces must be applied at the boundaries or in a layer of elements inside of boundaries**

**Several formulations available, e.g.:**

- Traction input (Zienkiewicz, Wolf)
- Domain Reduction Method (Bielak et. al.)



# Choosing the type of absorbing boundary

---

**Generally three types of boundary conditions**

- 1 Elementary boundary conditions
- 2 Consistent boundary conditions
- 3 Local boundary conditions

# Elementary boundaries does not satisfy radiation condition

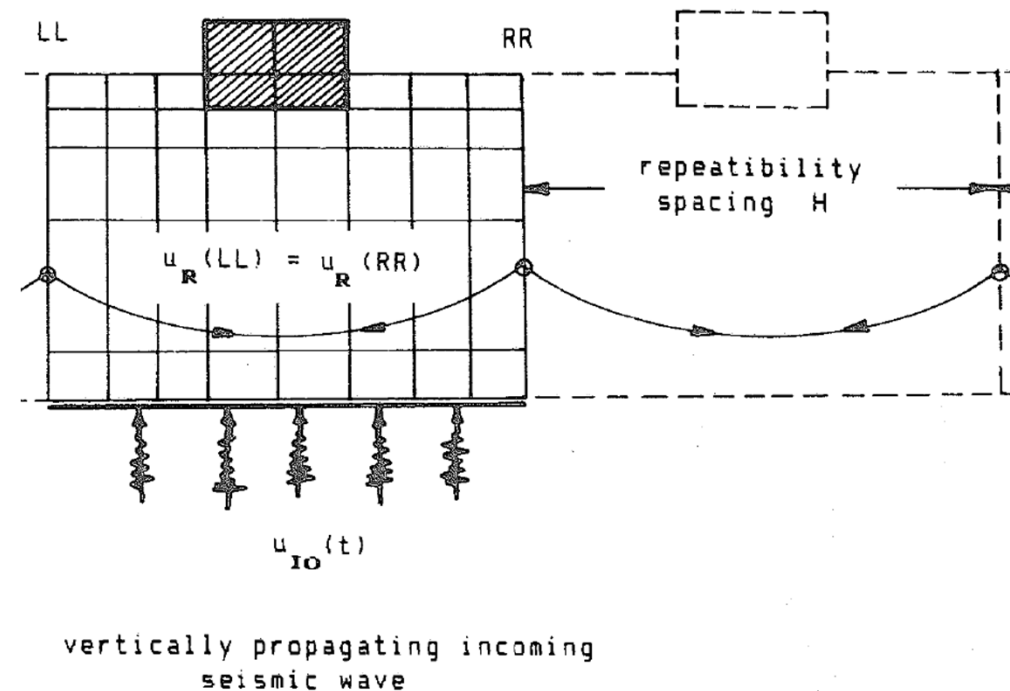
## 1 Elementary boundaries

Fixed (  $u = 0$  ) or free (  $\sigma = 0$  ) boundaries, or a combination

Does not satisfy radiation condition  
-> no energy absorption

“Tied” boundaries is a variation of elementary boundaries

- Sometimes used for soil analysis
- Enforces 1D conditions
- Not appropriate for stiff materials such as rock



Tied boundaries (Figure from Zienkiewicz, 1989<sup>1</sup>)

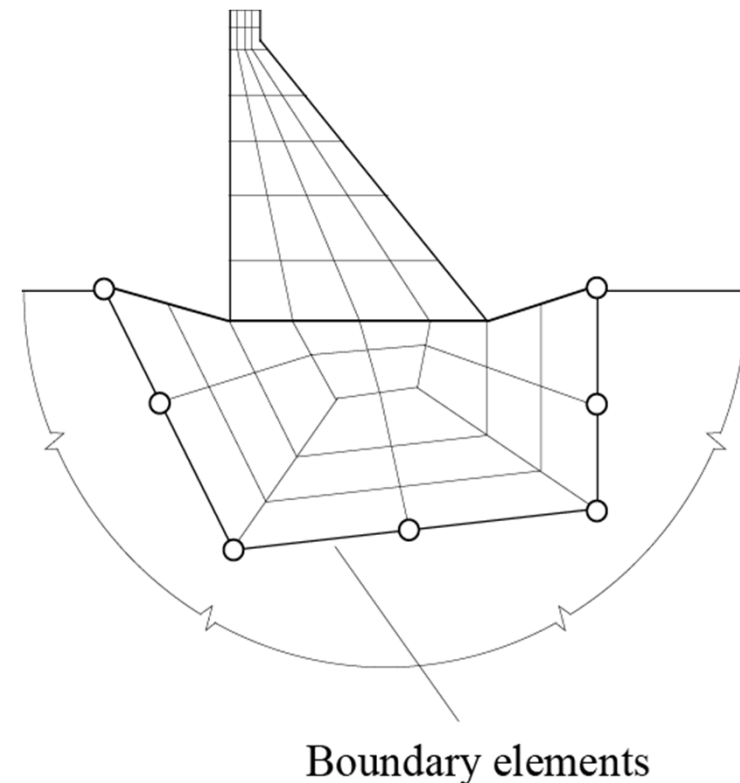
# Consistent boundaries are exact, but cumbersome to implement in time domain

## 2 Consistent boundaries

Global formulations that satisfy exactly the radiation condition

Frequency dependent formulations:

- e.g. Boundary Element Method
- Useful for steady-state problems
- Cumbersome to implement for transient time domain analyses



# Local boundaries most commonly used for transient time-domain analysis

## 3 Local boundaries

Local formulations that satisfy approximately the radiation condition

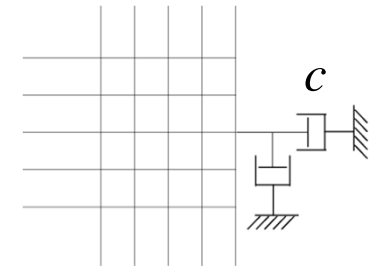
Error can be made sufficiently small by proper selection of parameters and domain size

Vast number of BCs available, e.g.:

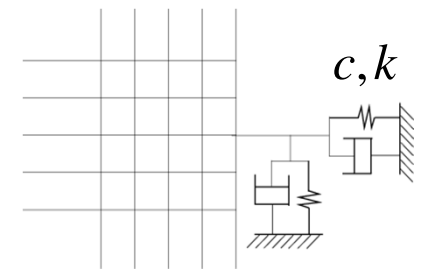
- Viscous dampers
- Viscous-spring (cone)
- PML
- Paraxial, Multi-Directional, Double-Asymptotic, +++

*Simple*

Viscous damper

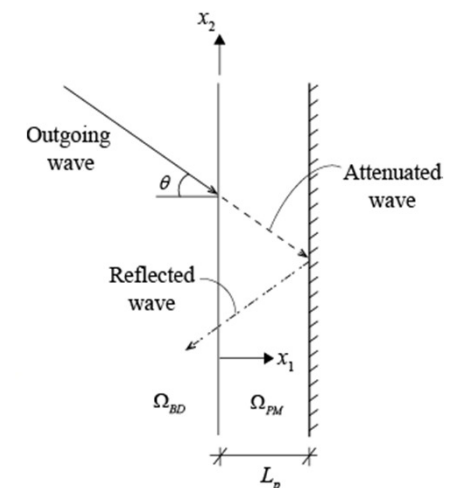


Viscous-spring (cone)



*Advanced*

Perfectly Matched Layer (PML)



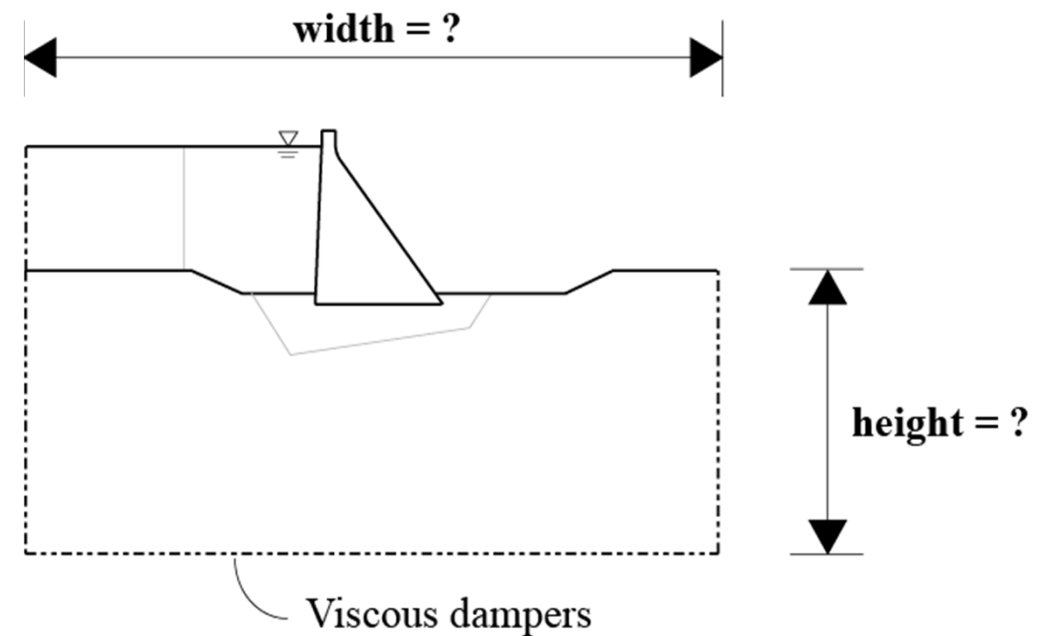
# Although among the simplest, the viscous damper is still most commonly used

**Viscous damper is accurate enough for most cases**

- But needs “large” domain sizes

**We selected to use the viscous damper for 2D / 3D analysis of concrete dams**

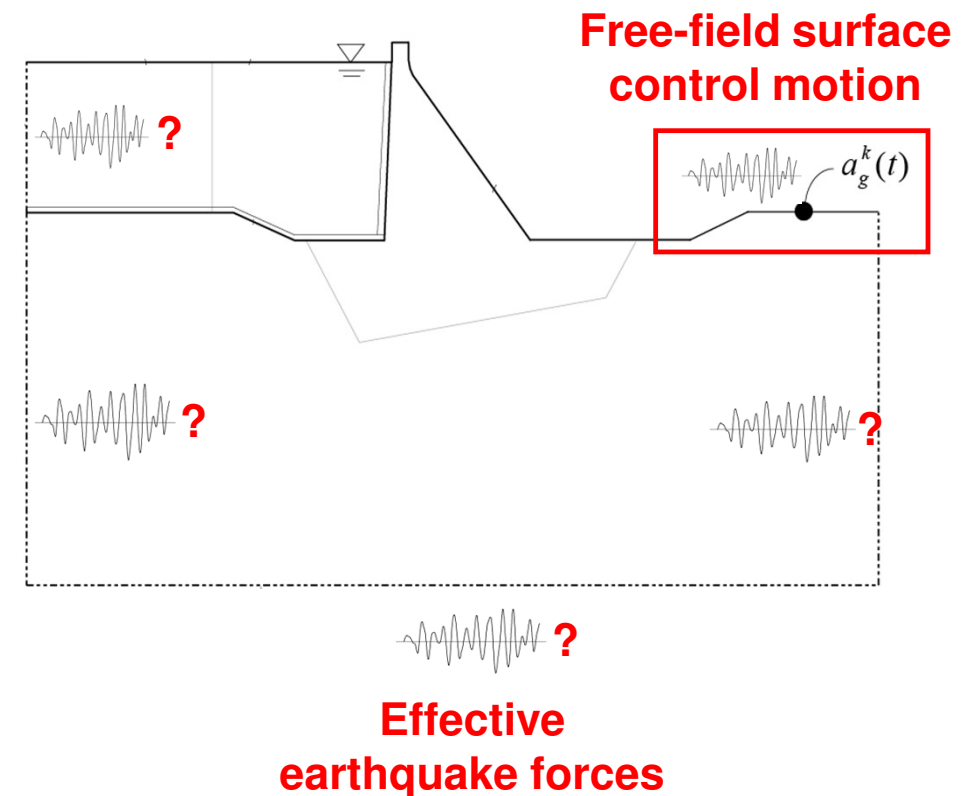
- Superior availability in FE codes



**The key question is:  
How big of a domain is required?**

## Second issue: How to specify the seismic input?

- Earthquake motion usually specified as free-field ground motion at the surface
- However, we need to apply forces (tractions) to the model to allow for absorbing boundaries

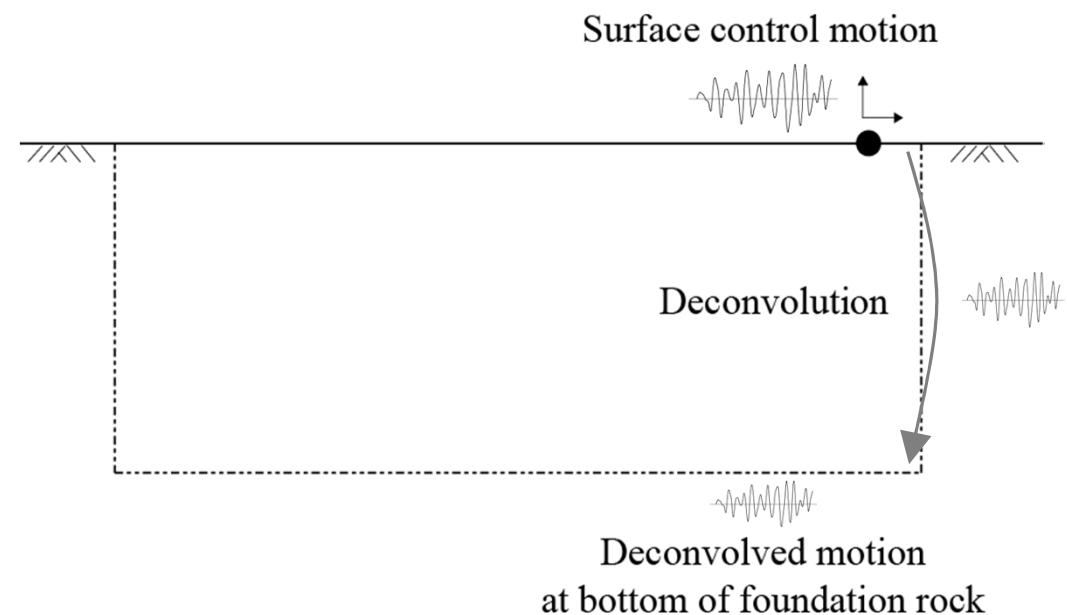


**Need a method to apply effective earthquake forces to the model**



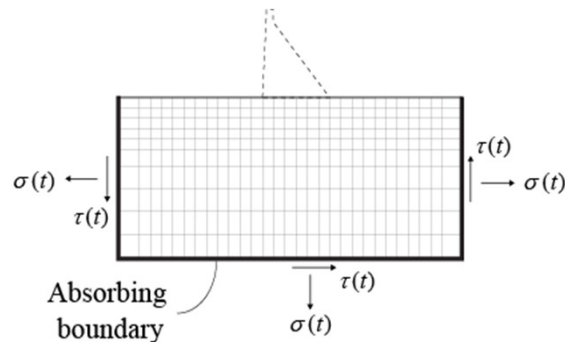
# Deconvolution to obtain free-field motion at depth

- Free-field control motion specified at ground surface
- Surface motion is deconvolved to obtain the motion at depth (assumes vertically propagating waves)
- If material damping in rock is neglected – can approximate deconvolution by taking  $\frac{1}{2}$  of surface motion



# Two types of methods are available to apply forces using free-field motion as input

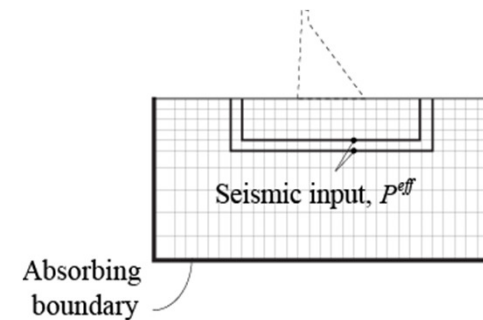
## Traction input method



$$\sigma(t) = \sigma^0(t) + \rho V \dot{u}^0(t)$$

- Forces applied directly to boundaries
- Formulation depends on the BC used
- “Only” vertically propagating waves
- Can be implemented using “free-field” boundary element (FLAC, Plaxis, Code\_Aster, ++)

## Domain Reduction Method

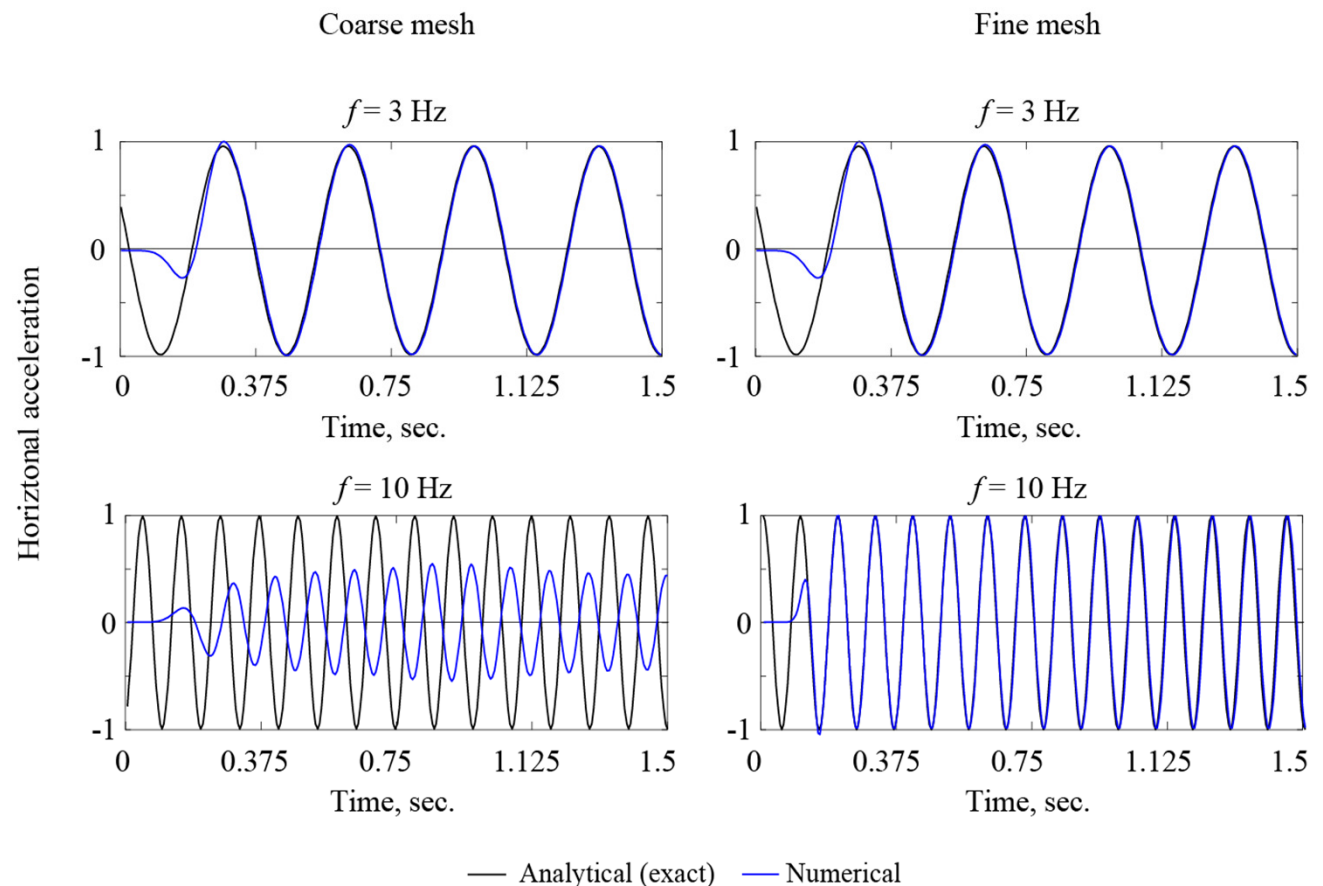
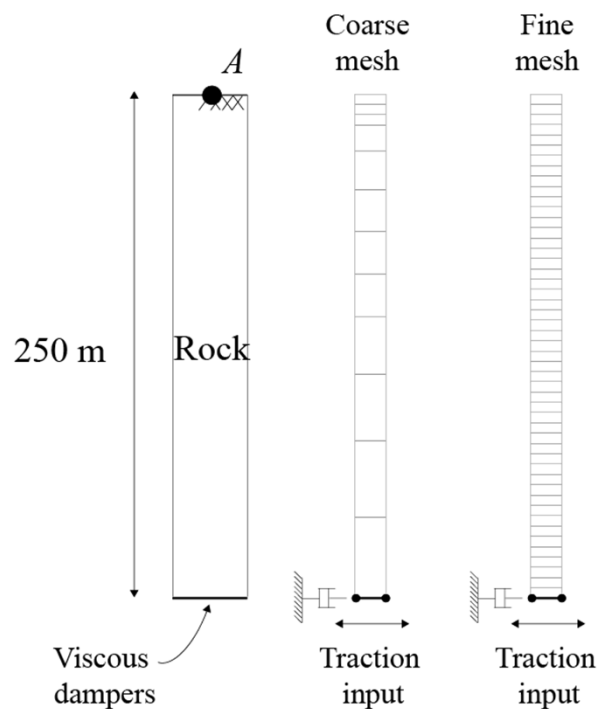


$$P^{eff} = \begin{Bmatrix} -M_{be}\ddot{u}_e^0 - C_{be}\dot{u}_e^0 - K_{be}u_e^0 \\ M_{eb}\ddot{u}_b^0 + C_{eb}\dot{u}_b^0 + K_{eb}u_b^0 \end{Bmatrix}$$

- Forces applied in single layer of elements
- Formulation decoupled from the BC used
- Can handle a full 3D wave field
- Clearly the most promising, but needs to be implemented in FE codes to be available (so far only LS-DYNA)

# Possible pitfall: Need to ensure sufficient mesh density to propagate high frequencies

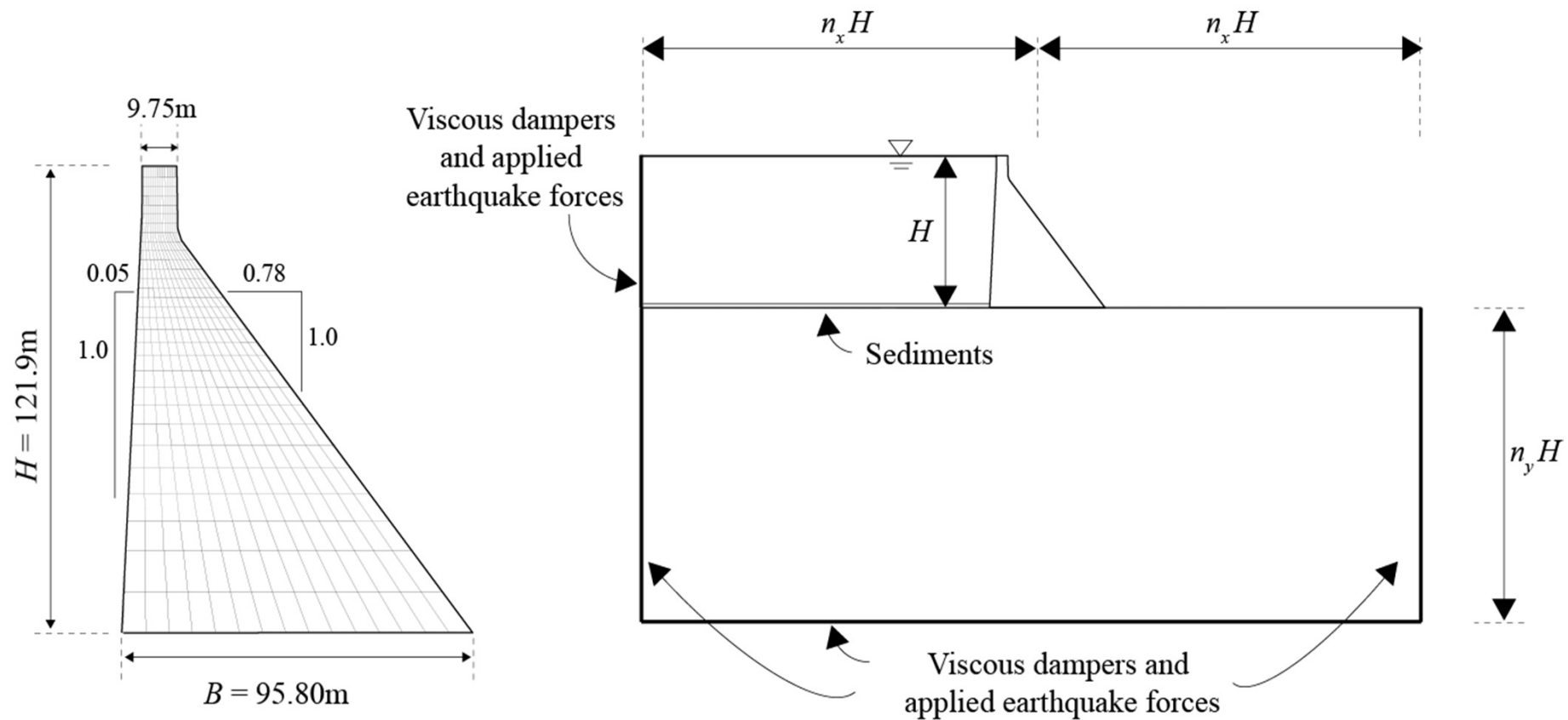
## 1D rock halfspace



**Should have 8-10 elements per shortest wavelength considered in analysis (Kuhlemeyer & Lysmer, 1973<sup>1</sup>)**

# **Response results for a gravity dam**

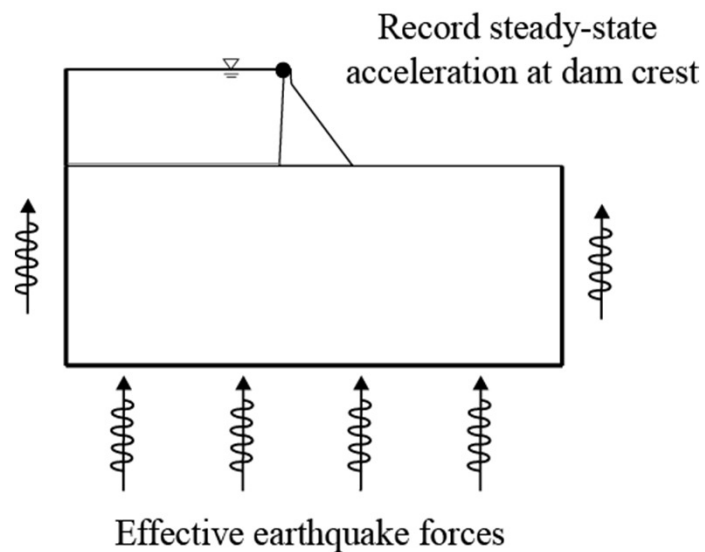
# Pine Flat Dam chosen for analysis



- **Geometry and material properties typical of concrete dams**
- **Repeated analysis for different domain sizes**
  - Investigate accuracy of viscous dampers
  - Semi-analytical solutions (EAGD-84) used as benchmark

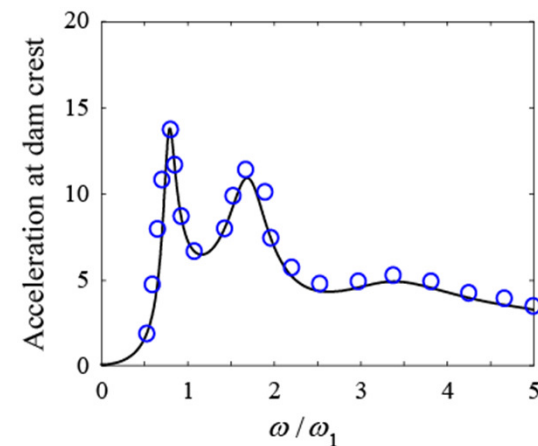
# Response results will be presented as frequency response functions for crest acceleration

## Analysis for single frequency



- Apply harmonic excitation as effective earthquake forces
- Compute steady-state acceleration at crest of dam

## Frequency response function



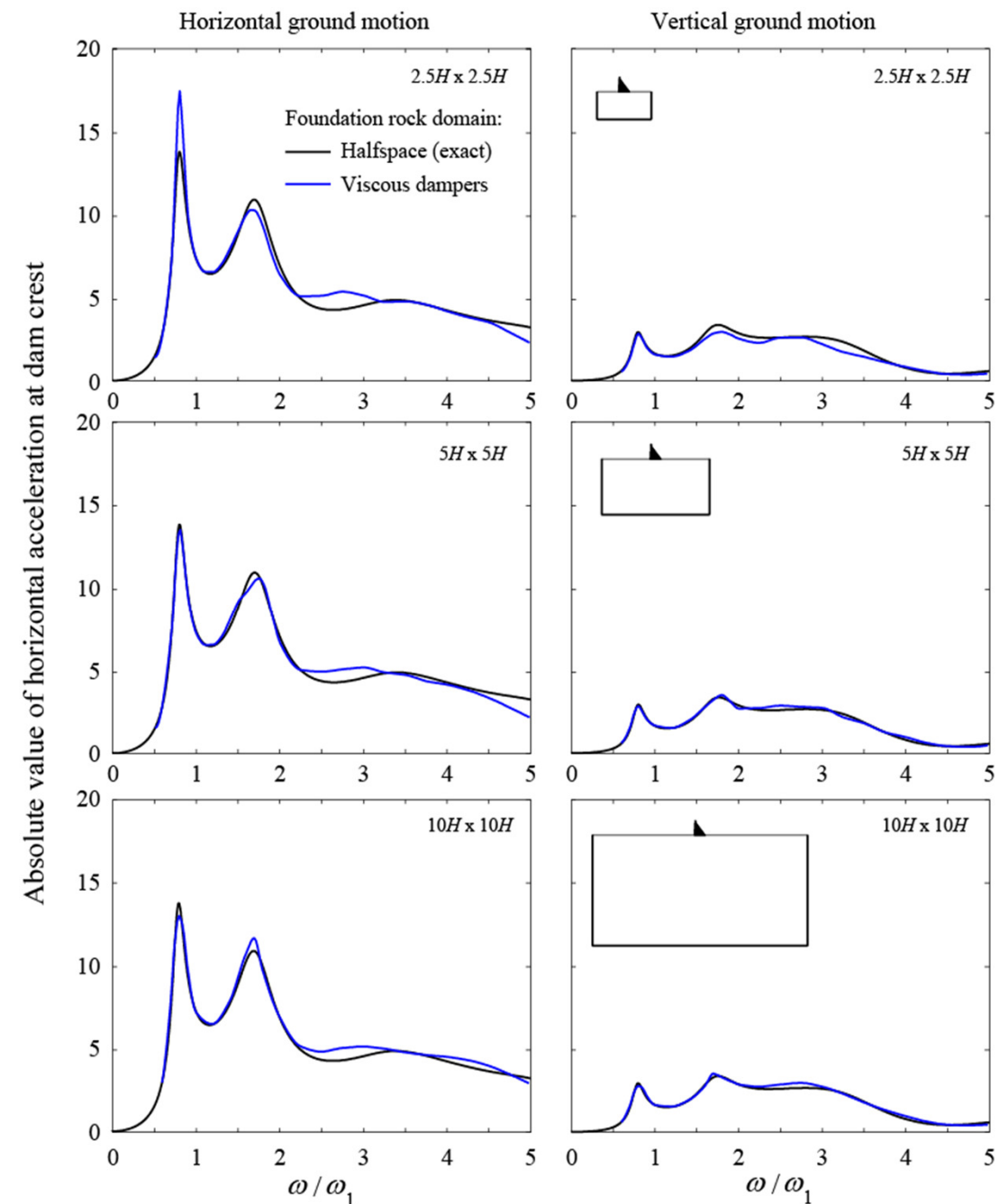
- Repeat procedure at sufficient number of frequencies to get a smooth plot
- FRF is a much more stringent test than comparing response to single EQ motion

# Effect of foundation-rock domain size on accuracy

*Dam on flexible rock, with empty reservoir*

## Better accuracy for larger domains

- As expected, results improve by increasing the domain size
- A size of  $5H \times 5H$  seems to ensure accurate results
- Increasing the size beyond  $5H$  seems to have little influence on the accuracy

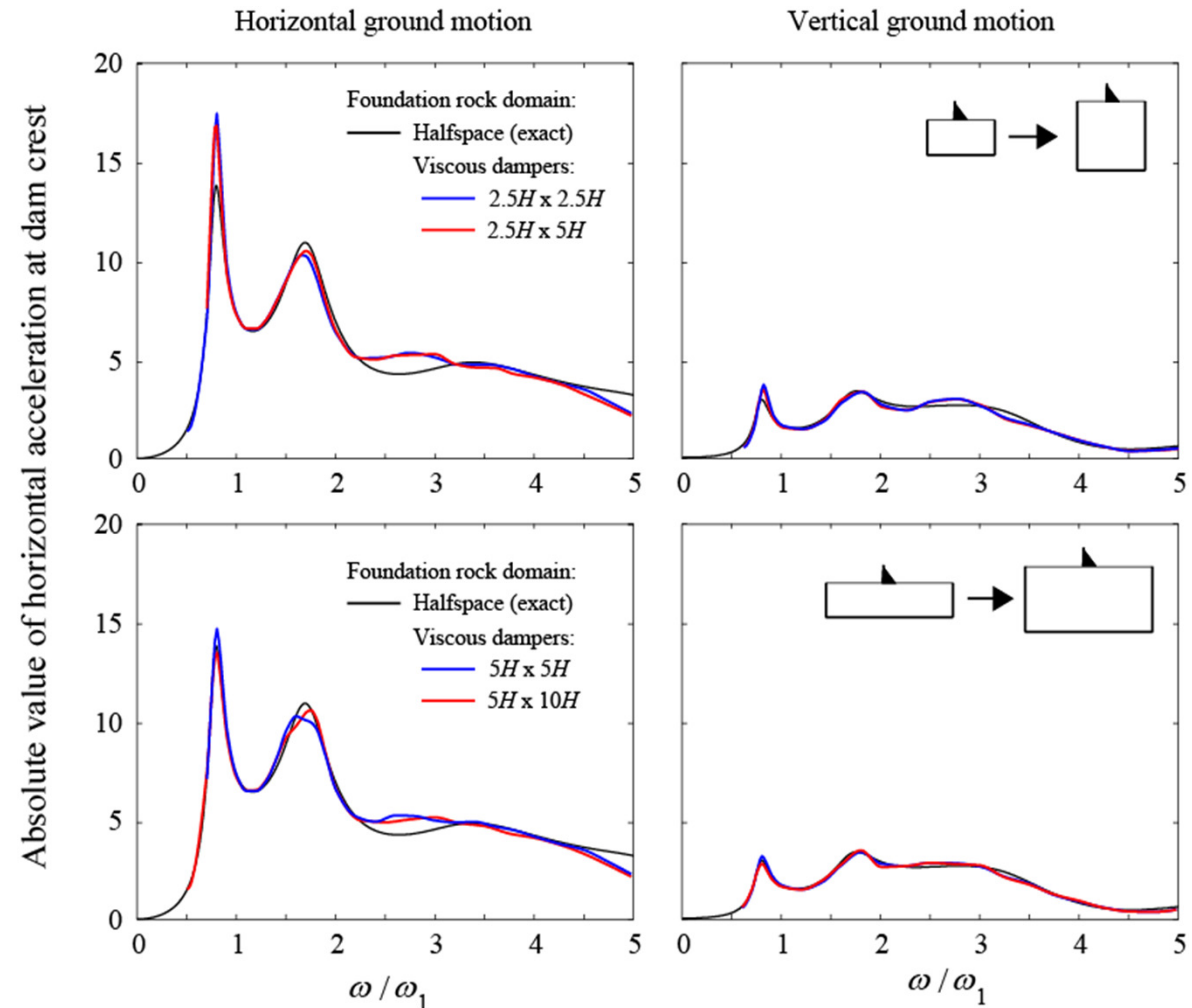


# Effect of aspect ratio of foundation-rock domain on accuracy

*Dam on flexible rock, with empty reservoir*

## The aspect ratio plays an important role

- Interestingly, increasing the depth of the model does not improve accuracy
- This occurs because dam creates a very shallow wave field – majority of energy in horizontal directions
- Much better “value” to add elements in the horizontal directions!



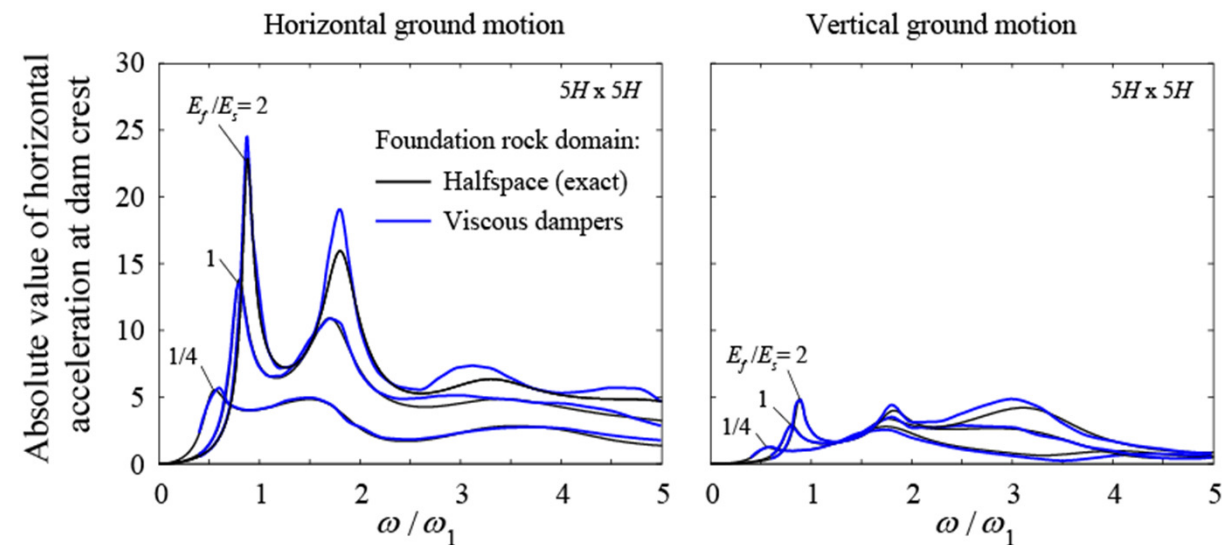


# Effect of foundation-rock stiffness on accuracy

*Dam on flexible rock, with empty reservoir*

**Softer rock tends to improve the accuracy of the results**

- **Counter-intuitive because interaction effects are more prominent for softer rock**
- **Occurs because viscous dampers performs better for waves with shorter wavelengths**

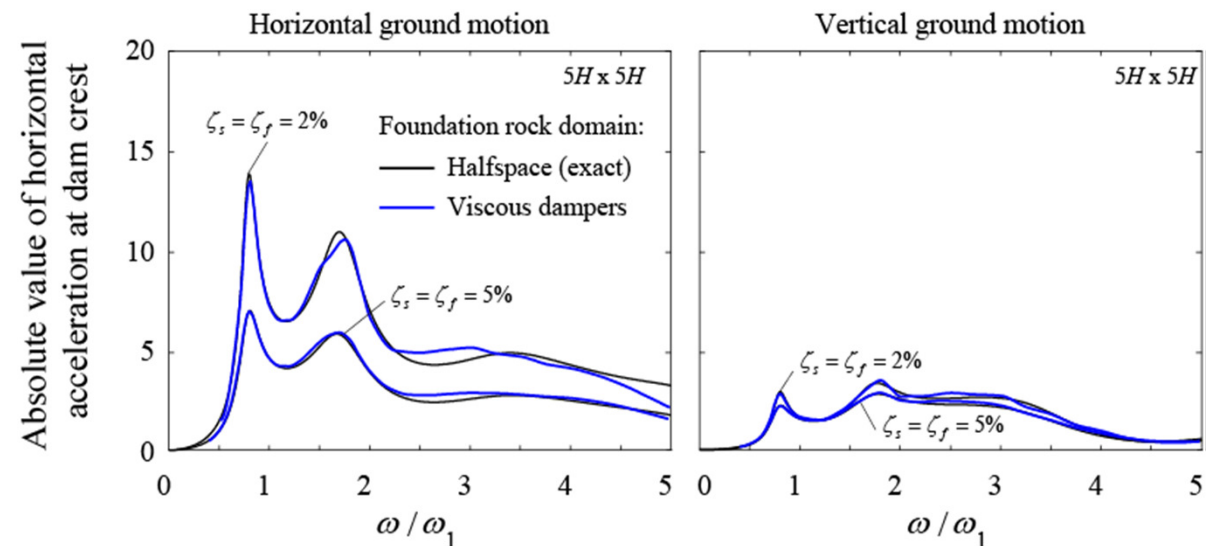


# Effect of material damping on accuracy

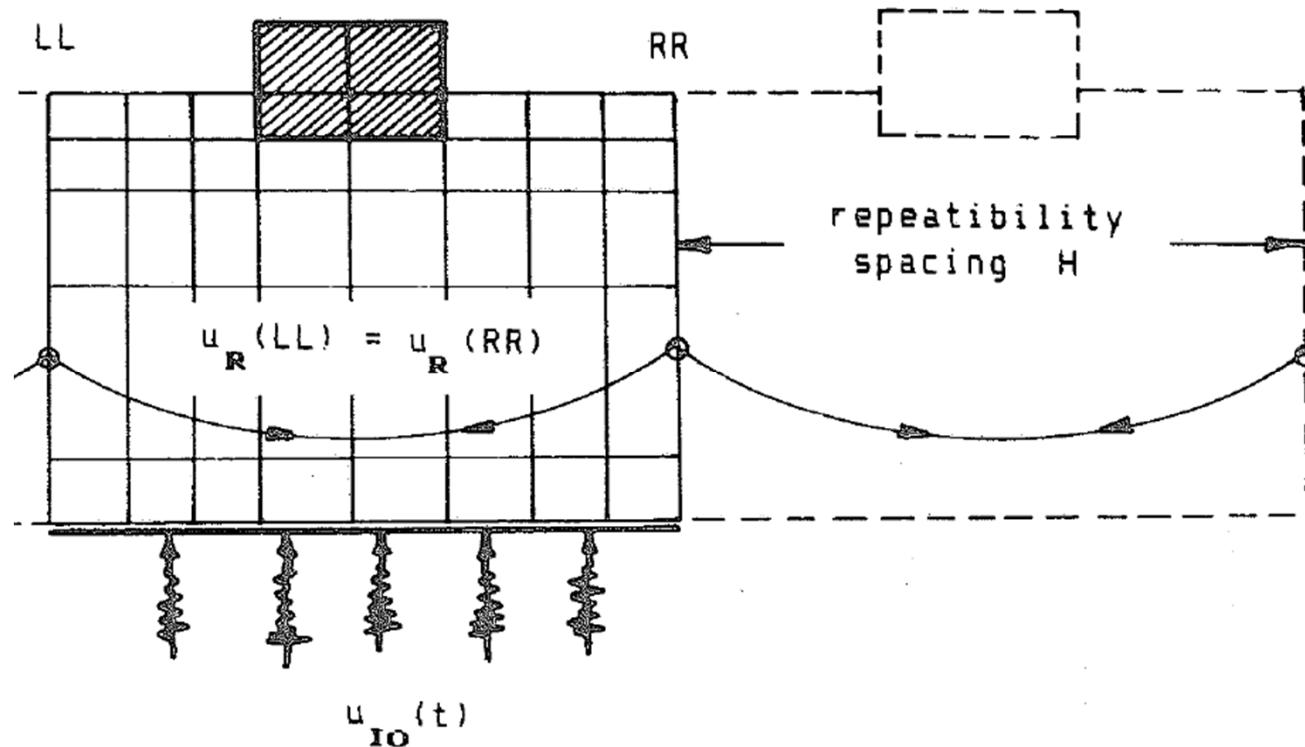
*Dam on flexible rock, with empty reservoir*

## Increased material damping improves accuracy slightly

- As expected, results improve by increasing the amount of material damping in the rock
- More of the waves are “damped out” before reflected to the dam
- Not a very prominent effect – cannot compensate for inadequate boundary by increasing material damping



# What if we ignore the absorbing boundaries altogether?



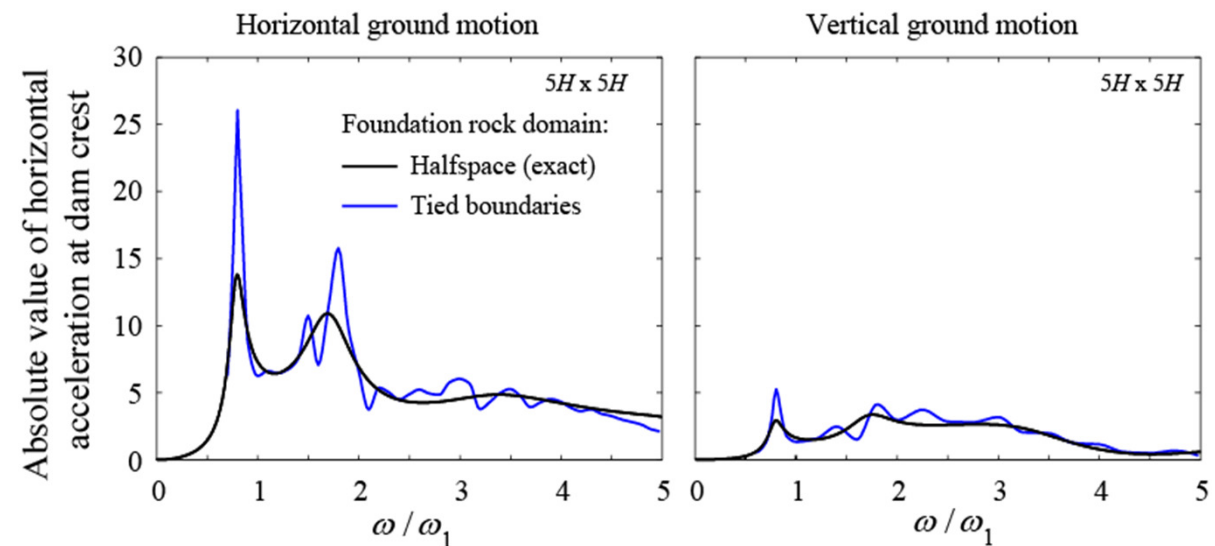
- Tied boundary model proposed by Zienkiewicz in 1989
- Popular for use in SSI analyses when modelling soft soils

# What if we ignore the absorbing boundaries altogether?

*Dam on flexible rock, with empty reservoir*

## Not an option to use tied boundaries for rock domains

- Results are in significant error when using tied boundaries
- Results are poor because no energy dissipation is allowed across the side boundaries

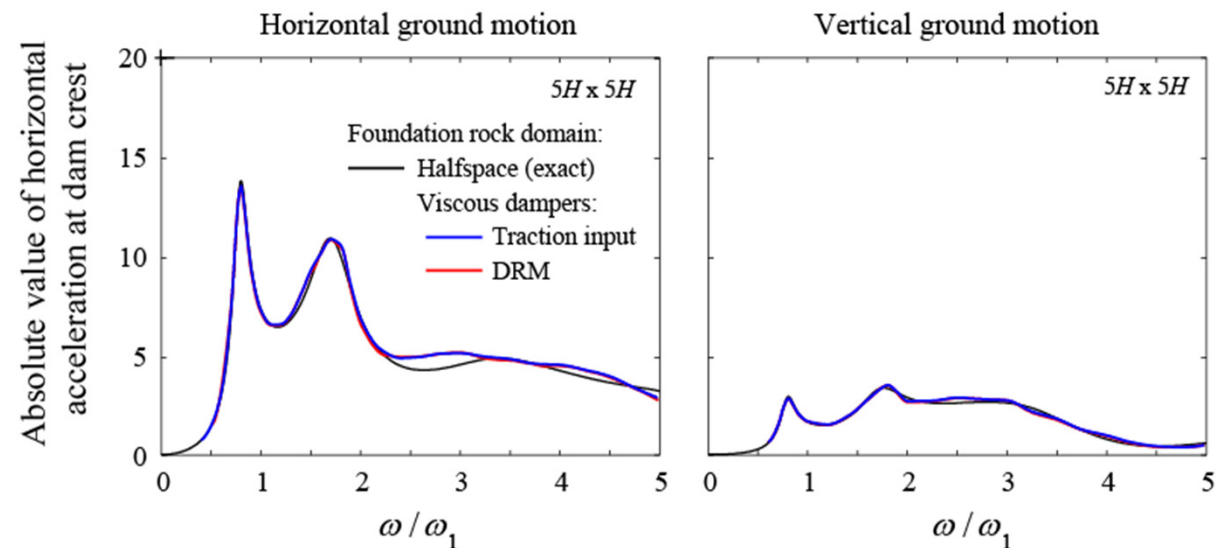


# Comparing methods for applying seismic input

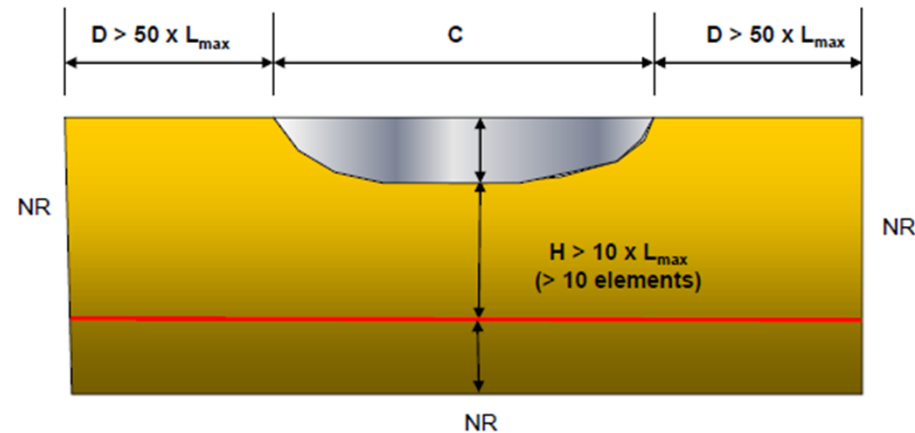
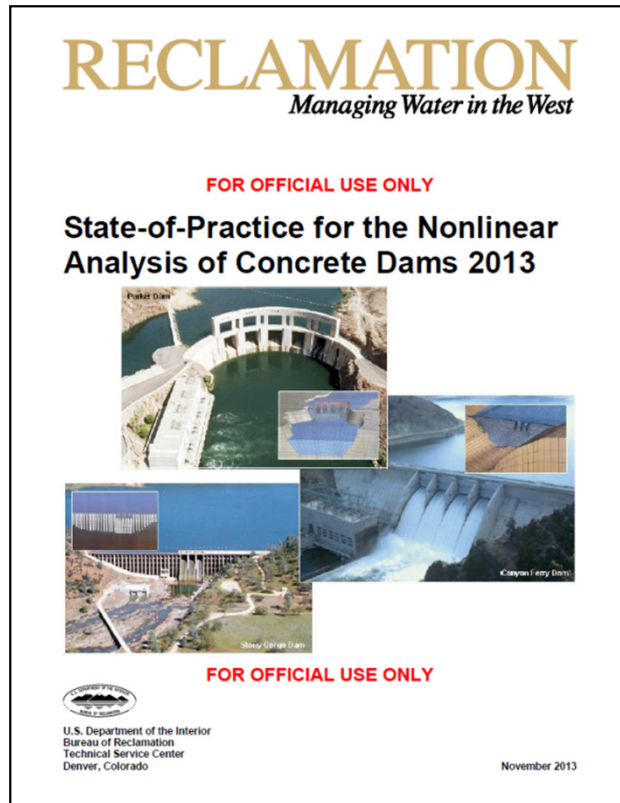
*Dam on flexible rock, with empty reservoir*

## DRM and traction input give identical results

- As expected, results are identical for the DRM and traction input method when implemented consistently



# What if we ignore forces on the side boundaries?



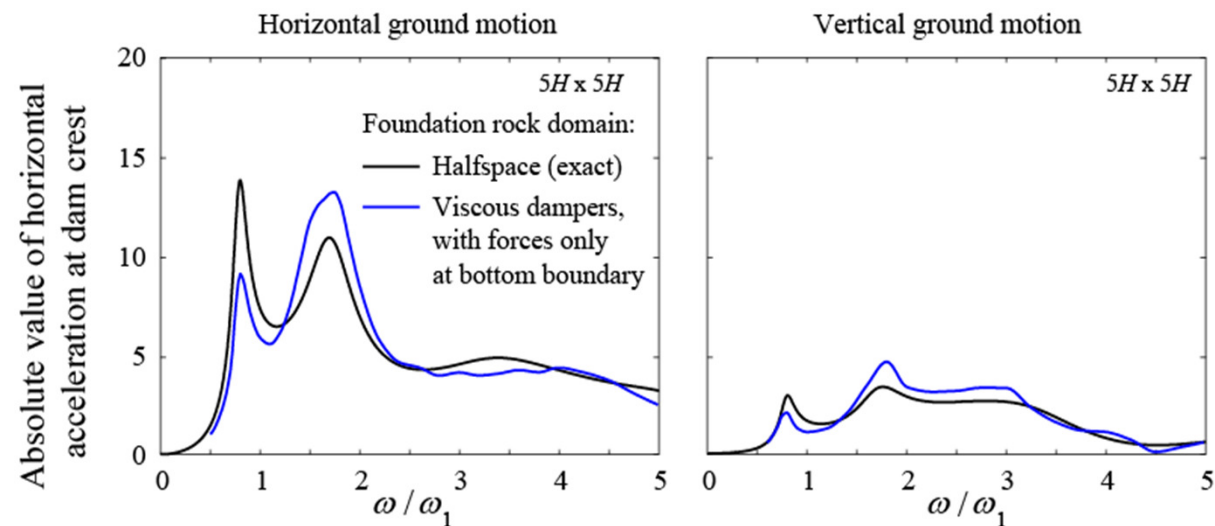
- Dam profession in the U.S. have tended to neglect forces at side boundaries
- Earthquake forces applied at single horizontal plane at depth in foundation-rock
- Model was never validated against substructure method

# What if we ignore forces on the side boundaries?

*Dam on flexible rock, with empty reservoir*

## Not an option to ignore forces on side boundaries

- Results are in significant error when ignoring forces at side boundaries
- Results are poor because side boundaries “drains” energy as the seismic waves propagates upward
- Attempts made to iterate on input motion to correct for this deficiency, but not clear that this will lead to acceptable results

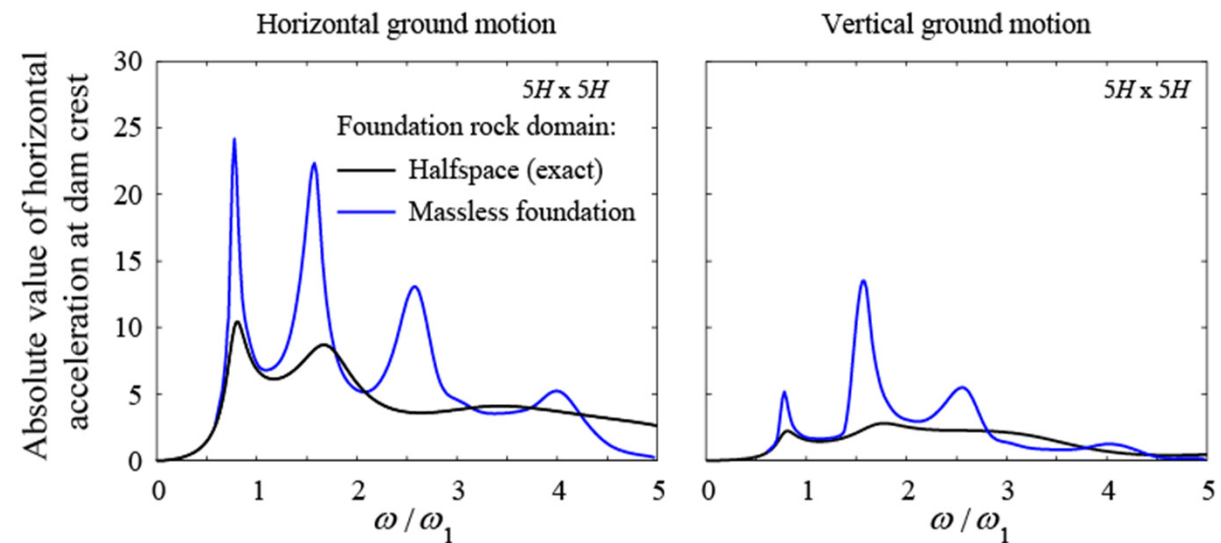


# What about the massless foundation model?

*Dam on flexible rock, with empty reservoir*

## Not an option to use the massless foundation model

- Results are unacceptable when using massless foundation model
- Results are poor because of total lack of energy dissipation in the foundation rock



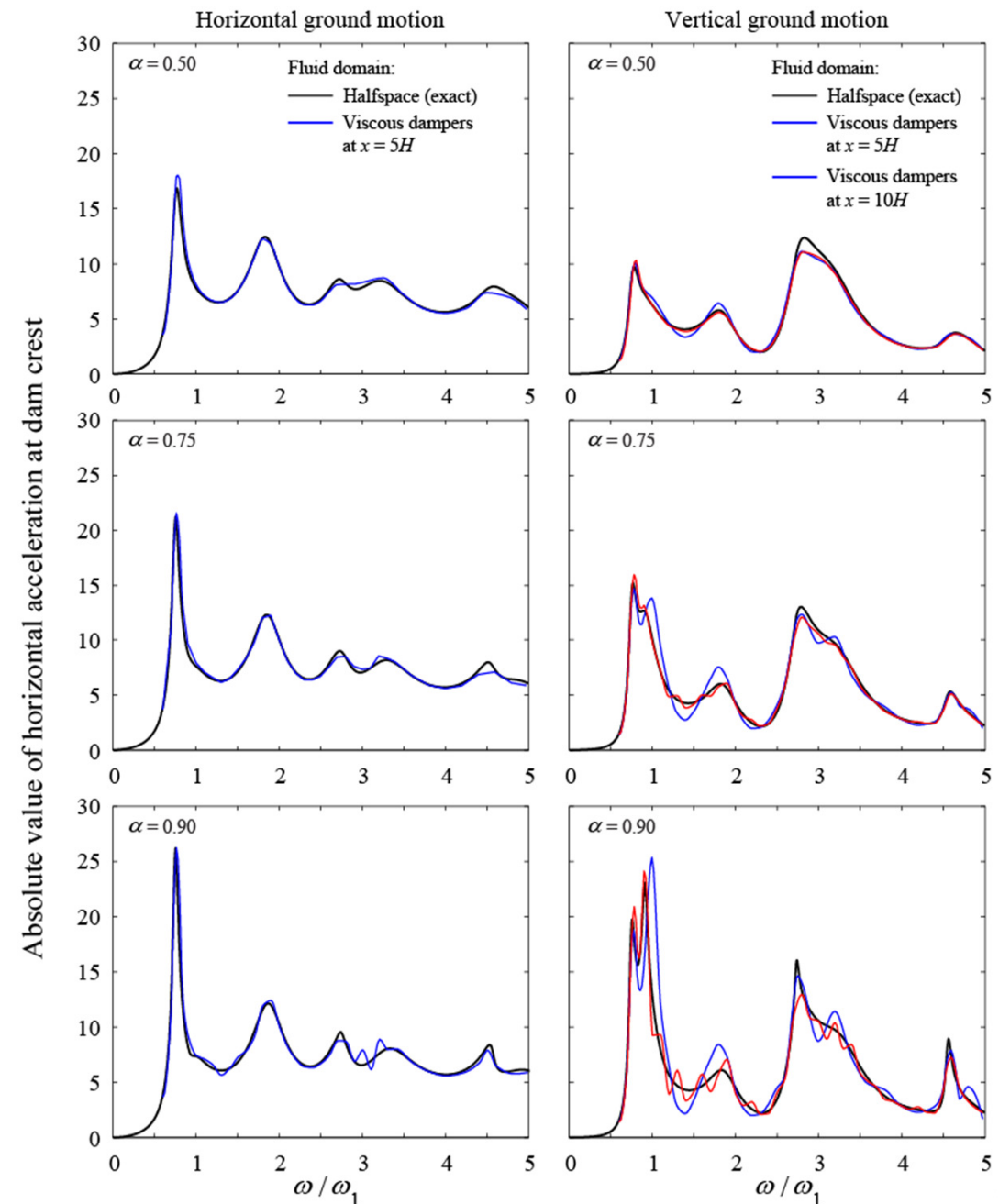
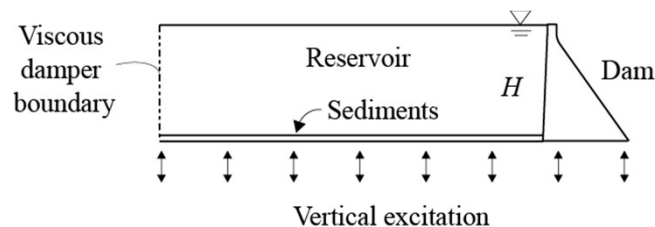


# Effect of fluid domain size on accuracy

*Dam on rigid rock, with full reservoir*

## Results are less sensitive than for foundation-rock domain

- A size of  $5H$  sufficient to ensure highly accurate results for most cases
- Scatter for vertical excitation is due to unrealistic assumptions for halfspace case (uniform motion cannot extend to infinity in u/s direction)



# Summary and recommendations

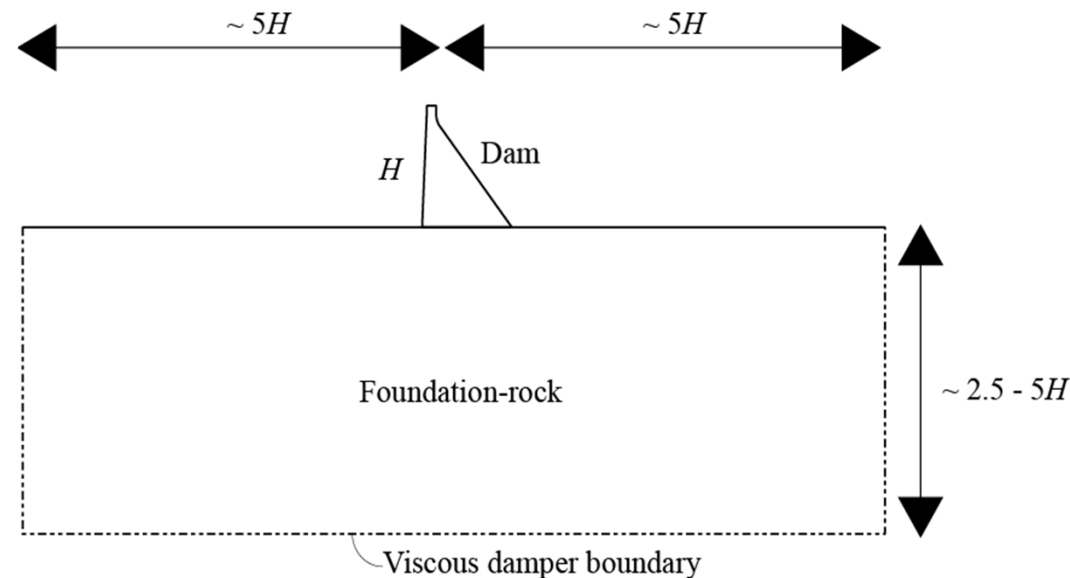
---

# Summary

## Foundation-rock domain

### Domain size of approx. $5H \times 2.5-5H$ should ensure accurate results

- With viscous dampers,  $5H$  in the horizontal and  $2.5 - 5H$  in the vertical direction should ensure accurate results for most analysis cases
- Aspect ratio matters – adding elements in the horizontal direction offers best “value for money”
- Need sufficient mesh density to ensure propagation of high frequencies (8 – 10 elements for smallest wavelength)



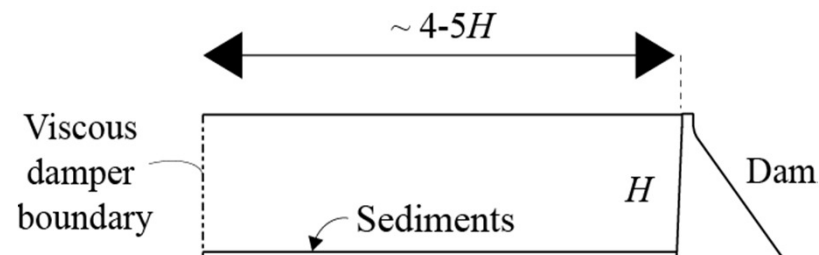
**Preliminary results indicate that narrower domains can be used for 3D systems (radiation in three directions)**

# Summary

## Fluid domain

### Domain size of approx. $4-5H$ should ensure accurate results

- Response of the dam is less sensitive to the size of fluid domain
- For viscous dampers, length of  $4-5H$  should ensure accurate results
- For practical analyses, the fluid domain size is most conveniently chosen as the same as the foundation-rock dimensions

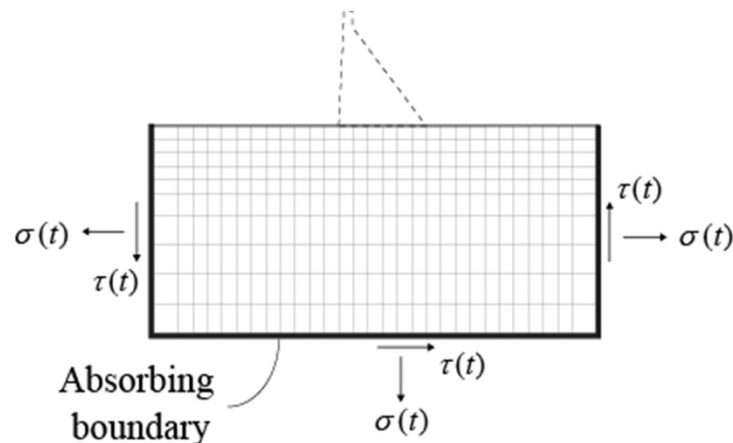


# Summary

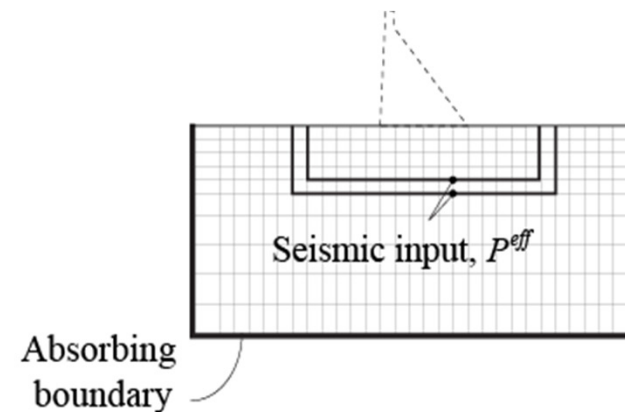
## Applying the seismic input

### Essential to apply the seismic input correctly

- Starting with a surface recorded free-field motion, two methods can be used to apply effective earthquake forces
  - Traction input method
  - Domain Reduction Methods (DRM)
- Should avoid using models with clearly demonstrated deficiencies (e.g. massless rock, or neglecting forces on side boundaries)



**Traction input**



**DRM**

# How can this be implemented for practical analyses?

## Unbounded domains

### Foundation rock domain

- Standard solid elements for rock
- Absorbing boundaries (such as viscous dampers) at truncations

### Fluid domain

- Acoustic elements are standard in most FE codes
- Absorbing boundaries (such as viscous dampers) at truncation
- Can use simple line elements to model reservoir bottom absorption

## Method for seismic input

### Automatic treatment inside FE code

- An increasing number of FE codes have “free-field” elements available (e.g. FLAC, Plaxis, Code\_Aster, ++)
- DRM formulation less available (LS-DYNA)

### Alternatively forces can be computed and applied to model independently of FE code

- However, this requires a bit of “book-keeping”, especially for 3D systems
- I’m currently developing a set of Matlab scripts to perform such tasks for an arbitrary 3D dam-water-foundation rock system

# Thank you for your attention!

---

*Arnkjell Løkke*

*Department of Structural Engineering  
NTNU*

**Contact:**

*E-mail:        [arnkjell.lokke@ntnu.no](mailto:arnkjell.lokke@ntnu.no)*

*Phone:        +47 48 04 88 43*