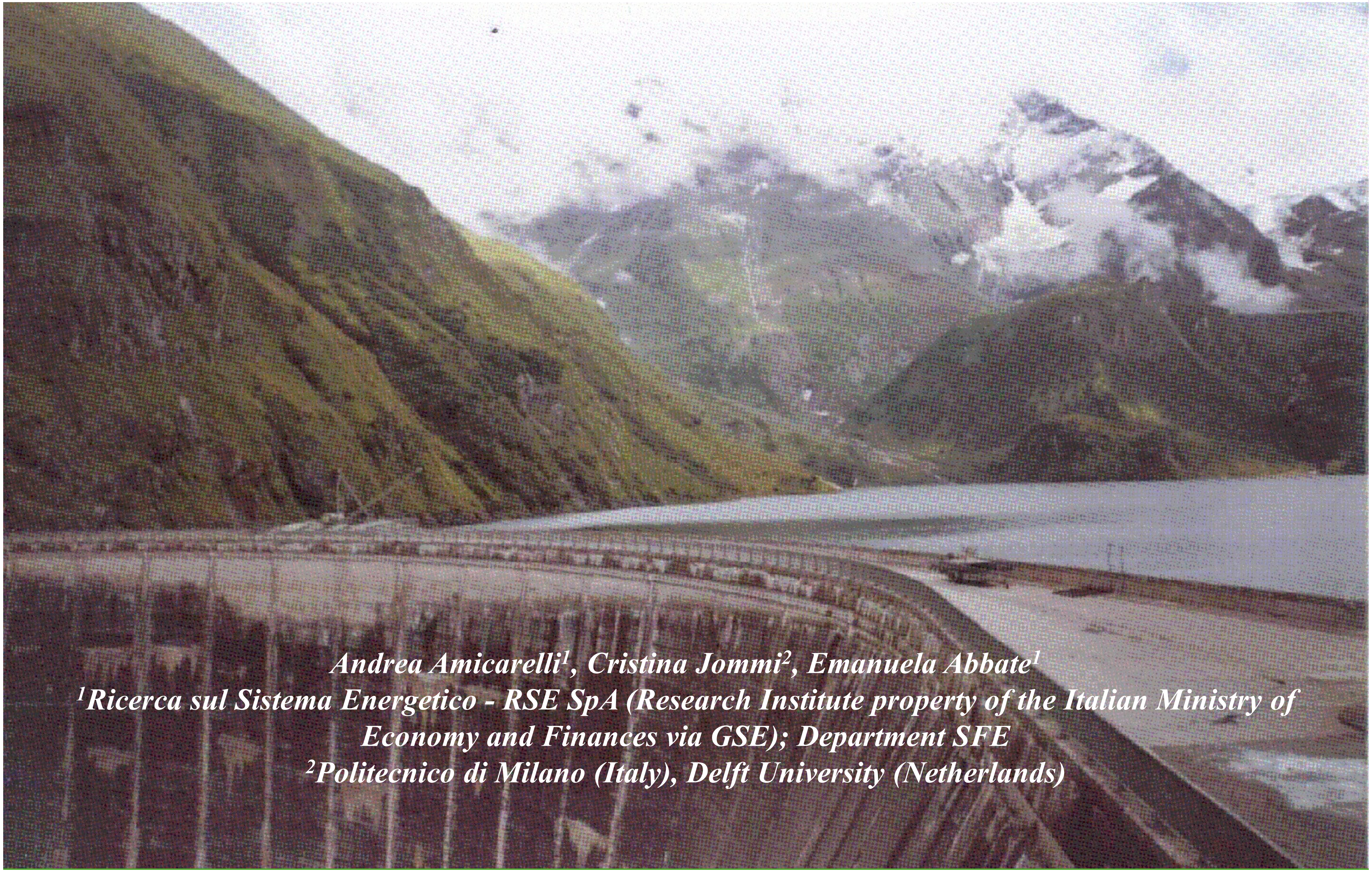
ICOLD 2019 (Milano, Italy, Politecnico di Milano; 9-10 September 2019)

SPH modelling of the Kagerplassen dyke failure



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Outline

1. **<u>Introduction</u>**
2. Reference mathematical and numerical models
3. Results (Kagerplassen dyke failure)
4. Conclusions

Introduction

RSE

- ## Main purposes
  - CFD (Computational Fluid Dynamics) simulation of 3D full scale flood-control works to secure them and preserve human health, the surrounding environment and anthropic activities/goods from flood-related damage
  - Preliminary demonstration on a 3D full scale dyke failure (run-out included)

- ## State-of-the-art
  - 3D CFD studies on dyke failures seem rare or absent in the reference literature

- ## Safety management of hydroelectric plants
  - Overtopping events; erosional dam breaks and dam breaches
  - Flood and landslide control works (protection devices): dikes, detention basins, flood control channels, longitudinal and transversal riverbed protection works (e.g. spur dikes, weirs), flood control barriers, soil pockets
  - Damage due to transport of solid bodies (e.g. structures, tree trunks, ice floes, vehicles)
  - Damage due to sediment transport (e.g., erosion/sedimentation; impact on power plants)
  - Riverine floods (urban flooding and black-out events)
  - Interaction of fast landslides with water bodies (reservoirs and watercourses)

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1. Introduction
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3D SPH mixture model for dense granular flows

Amicarelli et al. (2017, IJCFD)

- **<u>Development of a 3D SPH mixture model for bed-load transport and fast landslides: consistency with the “packing limit” of Kinetic Theory of Granular Flow (KTGF, e.g. Armstrong et al., 2010):</u>**

- **<u>State-of-the-art (numerical modelling of bed-load transport and landslides)</u>**
  - SPH (2D codes, 2-phase models or ad-hoc tuning for mixture viscosity): Farhadi et al. (2016), Wang et al. (2016), Grabe & Stefanova (2015), Pastor et al. (2015), Cascini et al. (2014), Wang & Chan (2014), Mabssout & Herreros (2013), Reyez Lopez et al. (2013), Ulrich (2013), Bui & Fukagawa (2013), Miao et al. (2012), Manenti et al. (2012), Capone et al. (2010), Qiu (2008).
  - SWE (FV or FD) models with ad-hoc tuning for the mixture viscosity or KTGF (2-phase).

3D SPH mixture model (continuity equation) Amicarelli et al. (2017, IJCFD)

- Continuity equations for the fluid («$_f$») and the solid («$_s$») phases (**KTGF**) and volume equation (ρ: density, ε: volume fraction, u_i: velocity component, Einstein notation for «$_j$»):

$$\frac{\partial(\rho_f \varepsilon_f)}{\partial t} = -\frac{\partial(\rho_f \varepsilon_f u_{f,j})}{\partial x_j} \qquad \frac{\partial(\rho_s \varepsilon_s)}{\partial t} = -\frac{\partial(\rho_s \varepsilon_s u_{s,j})}{\partial x_j} \qquad \varepsilon_s + \varepsilon_f = 1 \qquad \text{E.1}$$

- Definition of «mixture density» and «mixture velocity» (subscript «$_m$» omitted):

$$\boxed{\rho} \equiv \rho_f \varepsilon_f + \rho_s \varepsilon_s, \qquad \boxed{u_i} \equiv \frac{\rho_f \varepsilon_f u_{f,i} + \rho_s \varepsilon_s u_{s,i}}{\rho} \qquad \text{E.2}$$

- After summing the first two E.1 formulas, assuming **conservative particles** and then adopting a Weakly Compressible approach, the **continuity equation for the mixture** reads (continuum -left- and SPH approximation with SA-SPH BC -right-; «$_0$»: computational particle, «$_b$»: neighbouring particle, W: kernel, ω: particle volume, «$_w$»: wall, $\underline{n}$: normal, V_h’: kernel support completing volume):

$$\frac{d\rho}{dt} = -\rho \frac{\partial u_j}{\partial x_j} \qquad \frac{d\rho_0}{dt} = \rho_0 \sum_b (u_{b,j} - u_{0,j}) \left.\frac{\partial W}{\partial x_j}\right|_b \omega_b + 2\rho_0 \int_{V_h'} [(\underline{u}_w - \underline{u}_0) \cdot \underline{n}] n_j \frac{\partial W}{\partial x_j} dx^3 \qquad \text{E.3}$$

3D SPH mixture model (momentum equation) Amicarelli et al. (2017, IJCFD)

- Momentum equation for the fluid phase (**KTGF**; g: gravity acceleration, δ_{ij}: Kronecker delta, p: mixture/total pressure, τ_{ij}: shear/deviatoric stress tensor, K_{gs}: filtration coefficient):

$$\frac{d(\rho_f \varepsilon_f u_{f,i})}{dt} = -\delta_{i3} g \rho_f \varepsilon_f - \varepsilon_f \frac{\partial p_f}{\partial x_j} + \frac{\partial \tau_{f,ij}}{\partial x_j} - \boxed{K_{gs}(u_{f,i} - u_{s,i})} \qquad \text{E.4}$$

- Momentum equation for the solid phase (**KTGF**):

$$\frac{d(\rho_s \varepsilon_s u_{s,i})}{dt} = -\delta_{i3} g \rho_s \varepsilon_s \boxed{-\frac{\partial p_s}{\partial x_j}} + \boxed{\frac{\partial \tau_{s,ij}}{\partial x_j}} - \varepsilon_s \frac{\partial p_f}{\partial x_j} + \boxed{K_{gs}(u_{f,i} - u_{s,i})} \qquad \text{E.5}$$

- Mean effective stress (σ_m'):

$$\boxed{-\frac{\partial p_{s,m}}{\partial x_i}} = -\frac{\partial \sigma_m^{'}}{\partial x_i} \qquad \sigma_m^{'} \equiv \sum_{i=1}^{3} \frac{\sigma_i^{'}}{3} \qquad \sigma_m^{'} = \max\left(p - \max(p_f, 0), 0\right) \qquad \text{E.6}$$

- **Shear stress gradient term** in the momentum equation of the solid phase (Schaeffer, 1987; **«packing limit» of the KTGF**; μ_{fr}: frictional viscosity; e_{ij}: strain-rate tensor, φ: internal friction angle, I_2: second invariant):

$$\boxed{\frac{\partial \tau_{s,ij}}{\partial x_j}} = \frac{\partial}{\partial x_j}\left(2\mu_{fr} e_{ij}\right) \qquad \boxed{\mu_{fr}} \equiv \left(\frac{\sigma_m^{'}(\sin\varphi)}{2\sqrt{I_2(e_{ij})}}\right)$$

$$\mu_{fr} > \mu_{fr,\max} \Rightarrow \mu_{fr} \to \infty$$

Elasto-plastic regime (fixed particles) No influence of $\mu_{fr,max}$

3D SPH mixture model (momentum equation) Amicarelli et al. (2017, IJCFD)

- Mixture viscosity (H: Heaviside step function, $\varepsilon_{s,p}$=0.50 -KTGF-):

$$\boxed{\mu} \equiv \varepsilon_f \mu_f + H\left(\varepsilon_s - \varepsilon_{s,p}\right)\mu_{fr} \qquad \text{E.8}$$

- Consider: the sum of E.4+E.5; the volume equation; μ, ρ and u definitions; low spatial variation of $\mu_{fr;}$ rough linearization of phase velocities just for the computation of the shear stress gradient term: one obtains the **momentum equation for the mixture** (**same form as NS, but using mixture u_i, p, ν, ρ**):

$$\frac{du_i}{dt} = -\delta_{i3} g - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} \qquad \text{E.9}$$

- SPH approximation of the momentum equation for the mixture (SA-SPH BC, r: inter-element distance, ν_M: artificial viscosity, «$_{SA}$»: Semi-Analytic approach of Di Monaco et al., 2011, EACFM):

$$\left\langle \frac{du_i}{dt} \right\rangle_0 = -\delta_{i3} g + \frac{1}{\rho_0}\sum_b \left(p_b + p_0\right)\left.\frac{\partial W}{\partial x_i}\right|_b \omega_b + 2\frac{p_0}{\rho_0}\int_{V_h'} \left.\frac{\partial W}{\partial x_i}\right|_b dx^3 + 2\upsilon \sum_b \frac{m_b}{\rho_0 r_{0b}}\left(\underline{u}_b - \underline{u}_0\right)\left.\frac{\partial W}{\partial r}\right|_b + \qquad \text{E.10}$$

$$-\upsilon_M \sum_b \frac{m_b}{\rho_0 r_{0b}^2}\left(\underline{u}_b - \underline{u}_0\right)\cdot\left(\underline{x}_b - \underline{x}_0\right)\left.\frac{\partial W}{\partial x_i}\right|_b - \upsilon_M\left(\underline{u}_{SA} - \underline{u}_0\right)\cdot\left(\int_{V_h'} \frac{1}{r_{0w}^2}\left(\underline{x} - \underline{x}_0\right)\frac{\partial W}{\partial x_i} dx^3\right) + 2\upsilon\left(u_{w,i} - u_{0,i}\right)\cdot\left(\int_{V_h'} \frac{1}{r_{0w}}\frac{\partial W}{\partial r} dx^3\right)$$

3D SPH mixture model

Amicarelli et al. (2017, IJCFD)

(fluid pressure of saturated particles, time integration, EOS, multi-mixture simulations)

- Pressure of the fluid phase for saturated mixture particles, hypothesis of assuming local 1D filtration process parallel to the water table + stratified media (α_{WT}: slope angle of the water table, $\underline{n}_{sat\text{-}top}$: normal to the water table):

$$p_f = \rho_f g \left(z_{sat-top}\big|_{x_0, y_0} - z_0 \right) \cos^2\left(\alpha_{WT}\right) \qquad \alpha_{WT} = \max\left(\arccos\left(n_{sat-top,3}\right), \frac{\pi}{2} \right) \qquad \text{E.11}$$

- Stability criteria (h: kernel support size, Courant-Friedrichs-Lewy number CFL=0.1; C_ν=0.05) for the Leapfrog time integration scheme:

$$dt = \min_0 \left\{ C_\nu \frac{2h^2}{\nu}; CFL \frac{2h}{c + \left|\underline{u}\right|} \right\} \qquad \text{E.12}$$

- EOS (c: mixture sound speed):

$$p \cong c_{ref}^2 \left(\rho - \rho_{ref} \right) \qquad \text{E.13}$$

- Several media can be simultaneously modelled, if they belong to the following categories: **pure liquids** (ε_s=0, ε_f=ε_{liquid}=1), **dry soils** (ε_f=ε_{gas}), **saturated soils** (ε_f=ε_{liquid}).

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Geometry elaboration

- Benchmark definition (ICOLD 2019, Theme C):

ICOLD - Committee on Computational Aspects of Analysis and Design of Dams; 2019; "Theme C"; 15th Benchmark Workshop on Numerical Analysis of Dams (Milan, Italy), 9-11 September; https://www.eko.polimi.it/index.php/icold-bw2019/2019/about/editorialPolicies#custom-3

- Procedure to elaborate the geometries of the granular media and the water reservoir:
  - ☑ raw data digitization
  - ☑ roto-translation
  - ☑ elaboration of parametric curves
  - ☑ extrusion
  - ☑ surface mesh generation
  - ☑ format conversion

Geometry elaboration

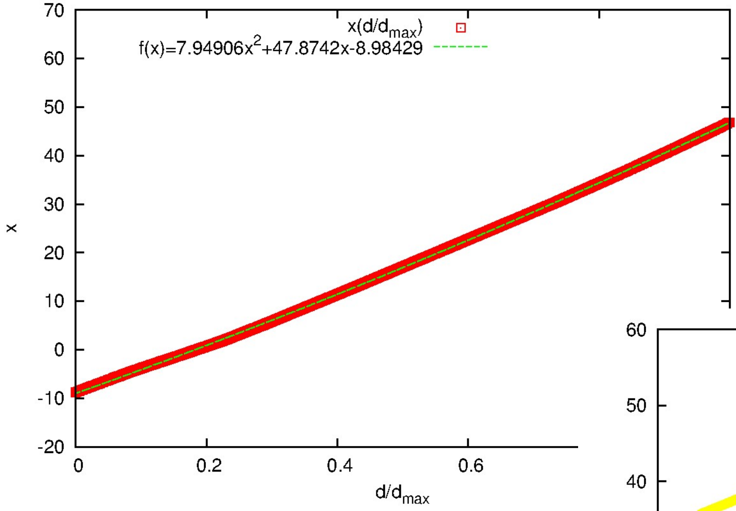
- Raw data digitization:
  - ☑ Engage Digitizer (Mitchell et al.), freeware
  - ☑ top view and the section “AA” are digitized (integration of 2nd and 3rd excavation)
  - ☑ top view map is georeferenced by means of the borehole positions

- 2D (horizontal) roto-translation
  - ☑ focus on two selected points, whose coordinates are available on both the maps: intersection between “section AA” and the coastline + intersection between “section AA” and the drill downstream edge (“least warped vector”)
  - ☑ Rotation (vertical section ref. syst. -> plan view ref. syst.) + origin translat.
  - ☑ Further, numerical ref. syst. is translated (with respect to the benchmark ref. syst. for the top views) by the offsets: $x_{off,1}$=101’200m; $y_{off,2}$=469700m

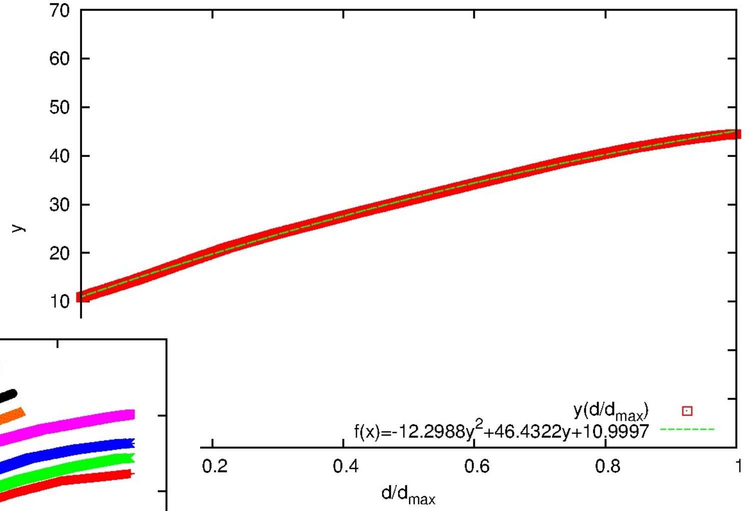
- Rotation angle (θ_R; $\underline{v}_A$ and $\underline{v}_B$: relative distance of the couple of selected points on both the reference systems):

$$\theta_R = ATAN_2(\sin\theta_R, \cos\theta_R) = \begin{cases} ATAN\left(\dfrac{\sin\theta_R}{\cos\theta_R}\right), & \cos\theta_R > 0 \\ \pi + ATAN\left(\dfrac{\sin\theta_R}{\cos\theta_R}\right), & \cos\theta_R < 0 \\ \dfrac{\pi}{2}, \quad \cos\theta_R = 0, & \sin\theta_R > 0 \\ -\dfrac{\pi}{2}, \quad \cos\theta_R = 0, & \sin\theta_R < 0 \end{cases} \qquad \cos\theta_R = \frac{\underline{v}_A \cdot \underline{v}_B}{|\underline{v}_A||\underline{v}_B|} \qquad \sin\theta_R = \frac{|\underline{v}_A \times \underline{v}_B|}{|\underline{v}_A||\underline{v}_B|} \qquad \text{E.1}$$

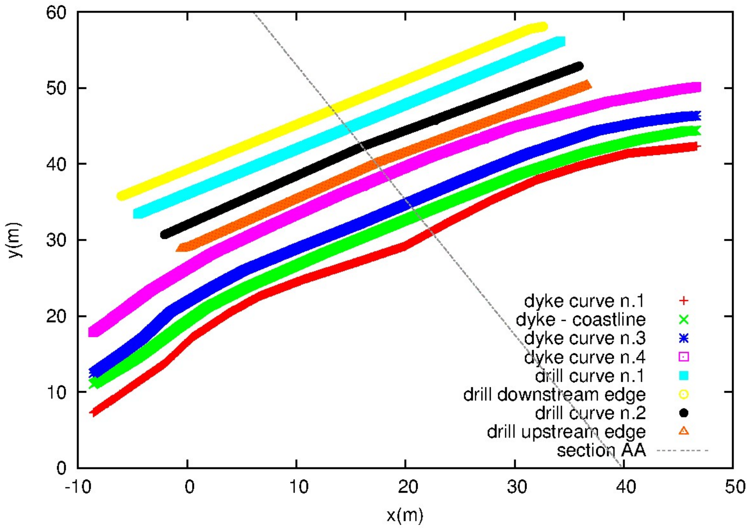
Digitized geometry data and regression parametric curves

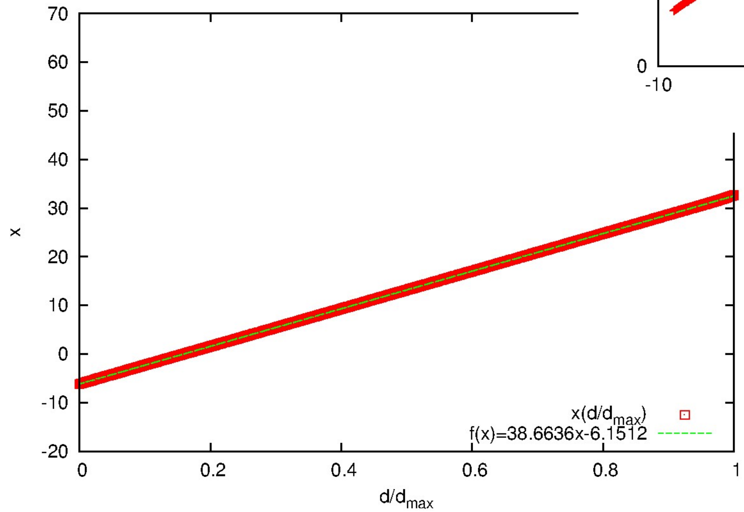


Dyke-coastline intersection; parametric curve $x(d/d_{max})$

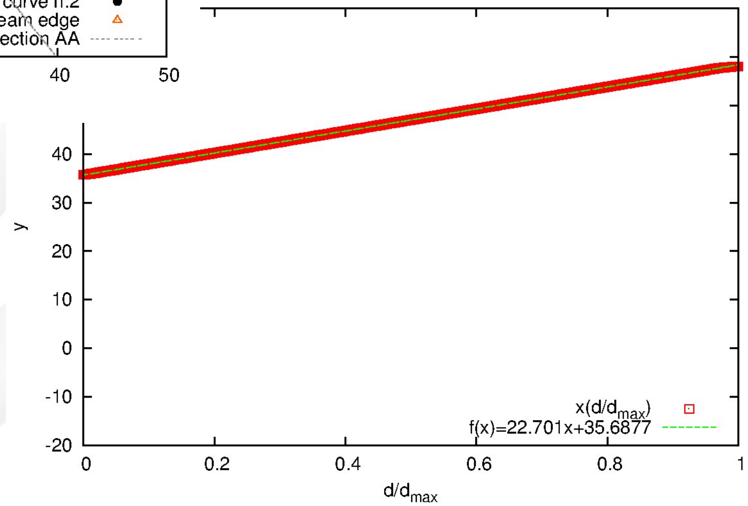


Dyke-coastline intersection; parametric curve $y(d/d_{max})$





Excavation downstream edge: parametric curve $x(d/d_{max})$



Excavation downstream edge: parametric curve $y(d/d_{max})$

Geometry elaboration

- Extrusion
  - ☑ each point of the section "AA" is used to describe a generic extruded curve in 3D
  - ☑ *x*/*y*-values of the points which discretize a generic 3D curve are linear interpolations of the two reference regression curves for *x*/*y* (coastline + drill downstream edge)

- Surface mesh generation (initial particle positioning grid)
  - ☑ 3D Delaunay triangulation + surface grid extraction (Paraview, Kitware et al.)

- Format conversion
  - ☑ ply2SPHERA_perimeter (RSE SpA): ".ply"-> SPHERA input format

Input data elaboration for the granular media

Table 1. Input quantities for the granular media.

Granular medium	φ(°)	γ(kN/m^3)	$e_{v,mean}$	ε_f	ρ_s(kg/m^3)
Dyke	30ca.	18.5	0.66	0.40	2'478
peat	40ca.	10.0	9.40	0.90	1'198
Top soil	30ca.	13.5	1.90	0.66	2'108
Organic silt/clay	20ca.	13.5	2.40	0.71	1'068

- Density of the solid phase (ρ_s) as function of porosity (ε_f), mixture specific weight (γ), and gravity acceleration (g):

$$\rho_s = \frac{\gamma}{(1-\varepsilon_f)g} - \rho_f \frac{\varepsilon_f}{(1-\varepsilon_f)} \qquad \text{E.1}$$

- Porosity as function of the void ratio (e_v):

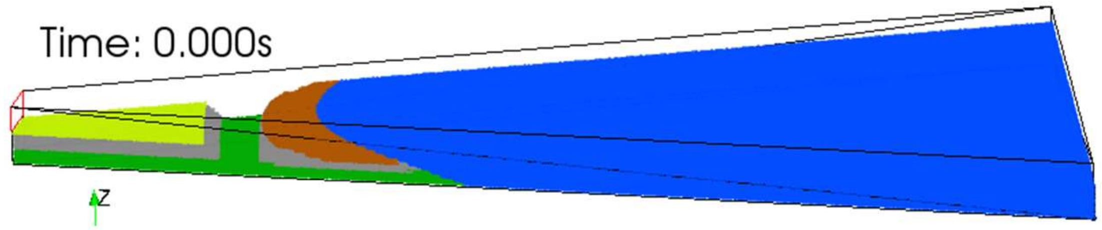
$$\varepsilon_f = \frac{e_v}{(1+e_v)} \qquad \text{E.2}$$

- Internal friction angle: values from geotechnical tests of Politecnico di Milano and Delft University

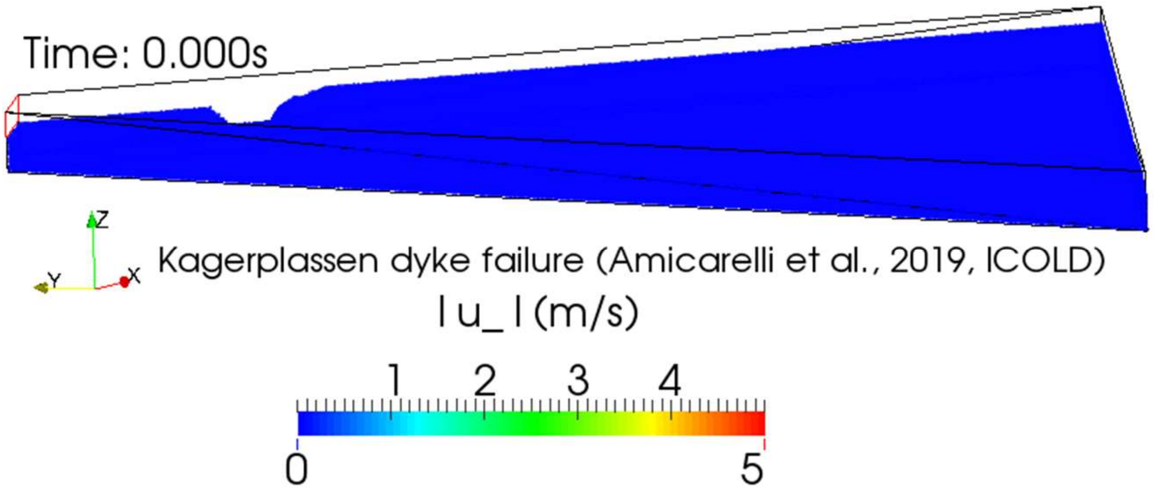
Test case configuration: other features

- ICs at the end of "excavation 3"; hydrostatic conditions dynamically imposed
- Open section at the downstream domain edge, above the top soil initial level
- Simulated time: 10s
- Spatial resolution: dx=0.5m; h/dx=1.3
- Stability criteria: CFL=0.05; C_v=0.05
- Preliminary results:
  - ☑ the limiting viscosity for the dyke is too low
  - ☑ no convergence analysis is carried out for the maximum viscosity
  - ☑ three media are featured by an imposed null kinematics (top soil, peat, organic silt/clay)
  - ☑ spatial resolution is relatively coarse
  - ☑ initial conditions for pressure are approximated
  - ☑ final time is reduced
  - ☑ Initial particle positioning grid has many elements of low quality and is not optimized
  - ☑ the actual internal friction angles are under revision
  - ☑ the probe H_4 is deactivated
  - ☑ the geometry of the symmetry planes are approximated
  - ☑ the role of vegetation is not considered
  - ☑ the configuration of the monitoring lines is not optimized

Velocity fields and medium interfaces

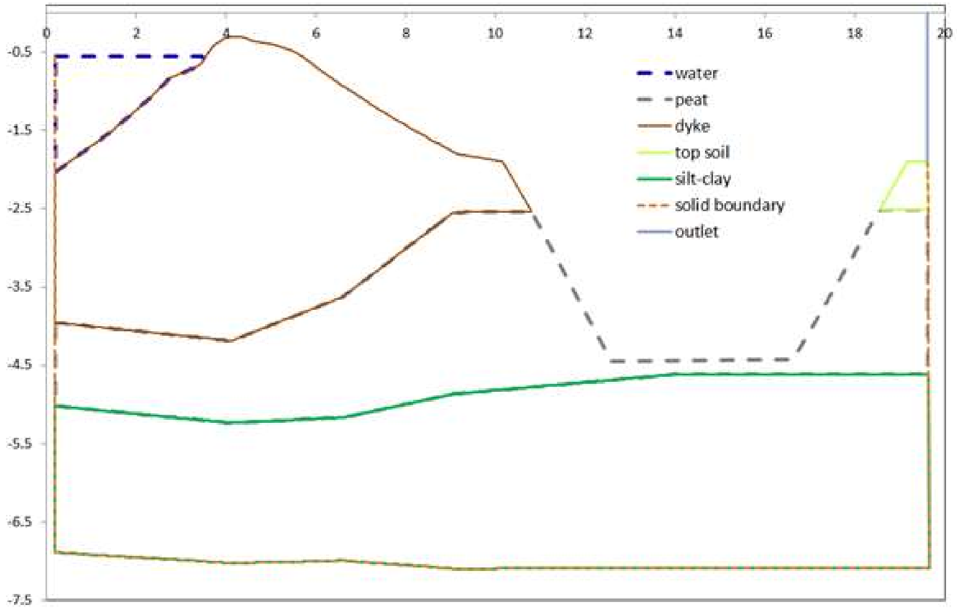


3D fields of the media
(blue: water; brown: dyke; grey: peat; yellow: top soil; green: organic silt/clay)

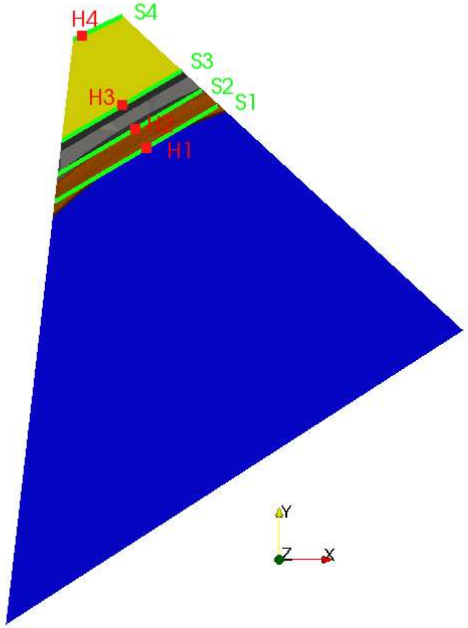


3D fields of the absolute value of velocity

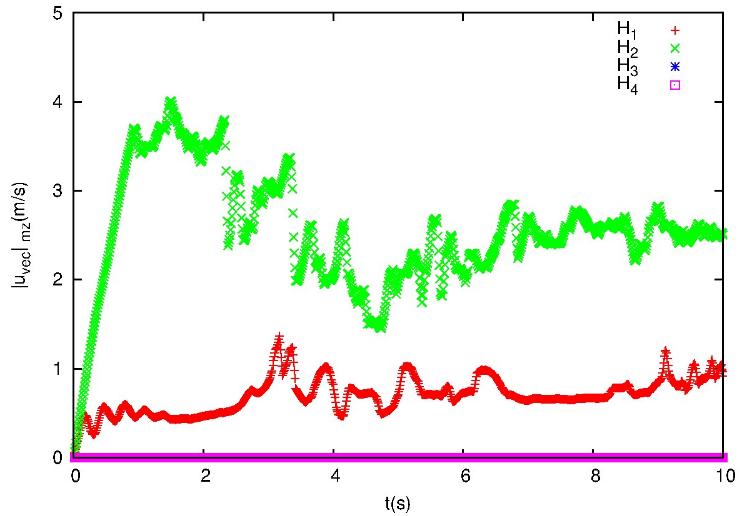
Simulated time series: velocity, fluid top height



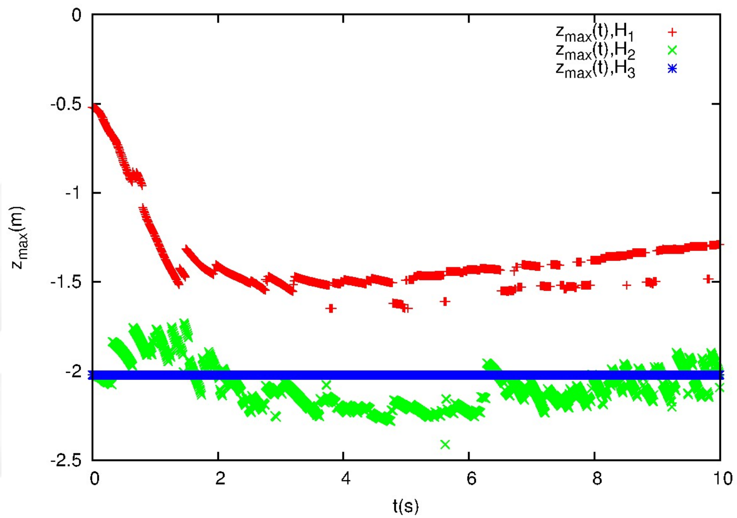
2D variant (medium ICs)



Monitoring lines (H_i) and sections (S_i)

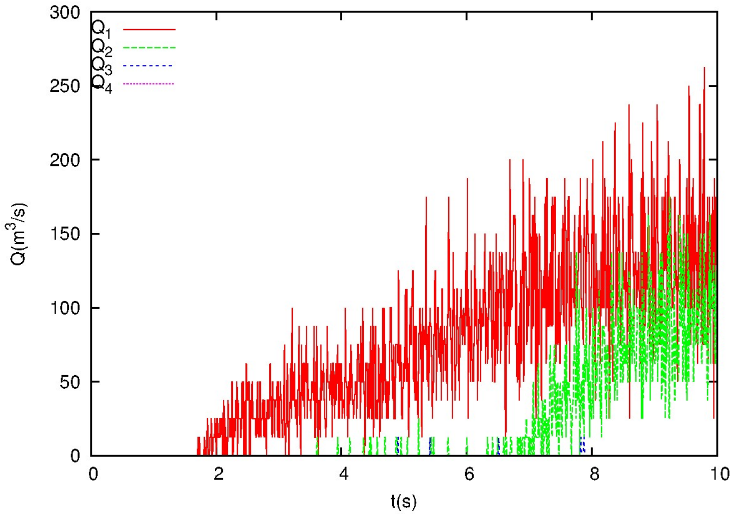


Absolute value of velocity

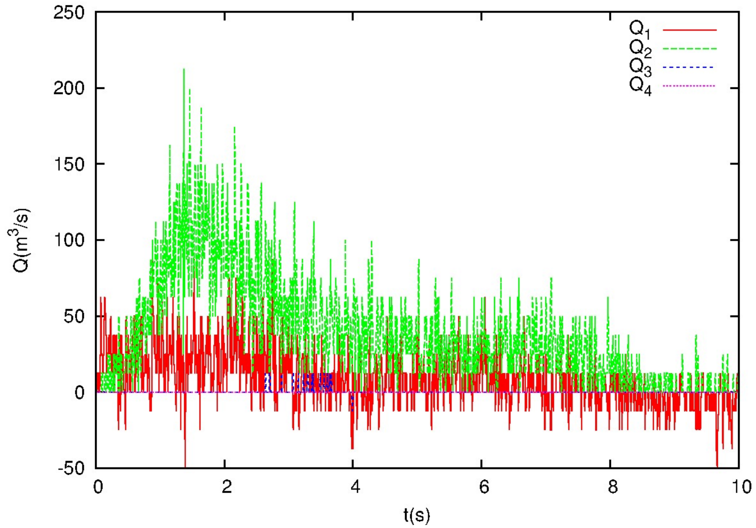


Fluid top height

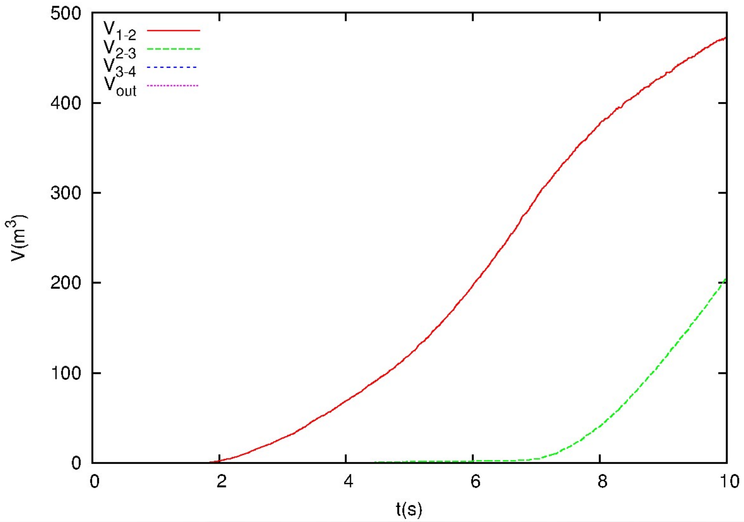
Simulated time series: flow rates, cumulated volumes (sub-domains)



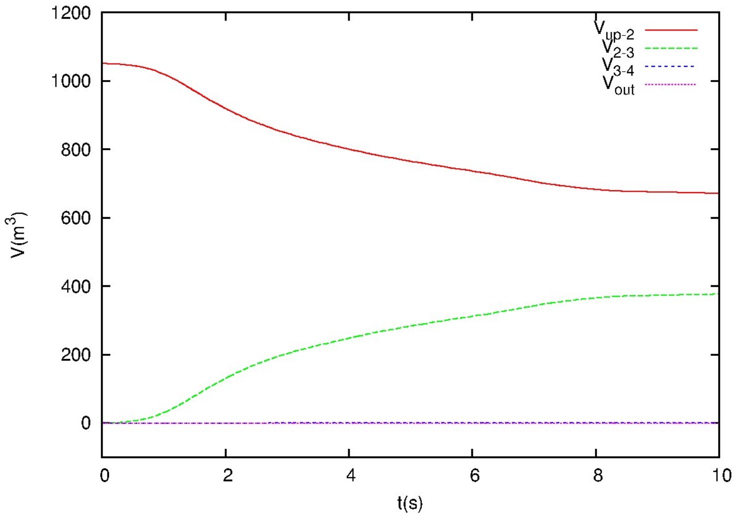
Flow rate hydrographs for water



Flow rate hydrographs for the dyke saturated granular material



Cumulated volumes for water



Cumulated volumes for the dyke saturated granular material

Conclusions

1. CFD simulation of 3D full-scale Kagerplassen dyke failure on-site experimenton (demonstrative test with preliminary results)
2. 3D geometries (granular media and water reservoir) are reconstructed from the available measures by means of an analytical procedure
3. SPH results are provided in terms of: 3D CFD fields (medium interfaces and velocity); hydrographs (time series) for fluid top height, flow rate, cumulated volumes and velocity
4. SPHERA dynamically simulates: triggering and initial propagation of the sliding surfaces within the dyke + whole dyke failure + landslide run-out + water flood
5. A 2D variant of the dyke failure is under investigation in terms of: 3D effects; CPU time; memory allocation
6. A list of possible improvements is reported to provide a more detailed description of the preliminary results of this on-going study
7. **SPHERA v.9.0.0 (RSE SpA) is a** CFD-SPH **FOSS (Free/Libre & Open Source Software)** code distributed on a public **GitHub repository** (github.com). Applications: floods (with transport of solid bodies, bed-load transport, domain spatial coverage up to some hundreds of squared kilometres, damage scheme for electrical substations, flood-control works), landslides and wave motion, hydroelectric plants, fuel sloshing tanks, hydrodynamic lubrication.

SPHERA v.9.0.0 (RSE SpA)

Github repository

(https://github.com)

Main features of SPHERA (certified on IJs):

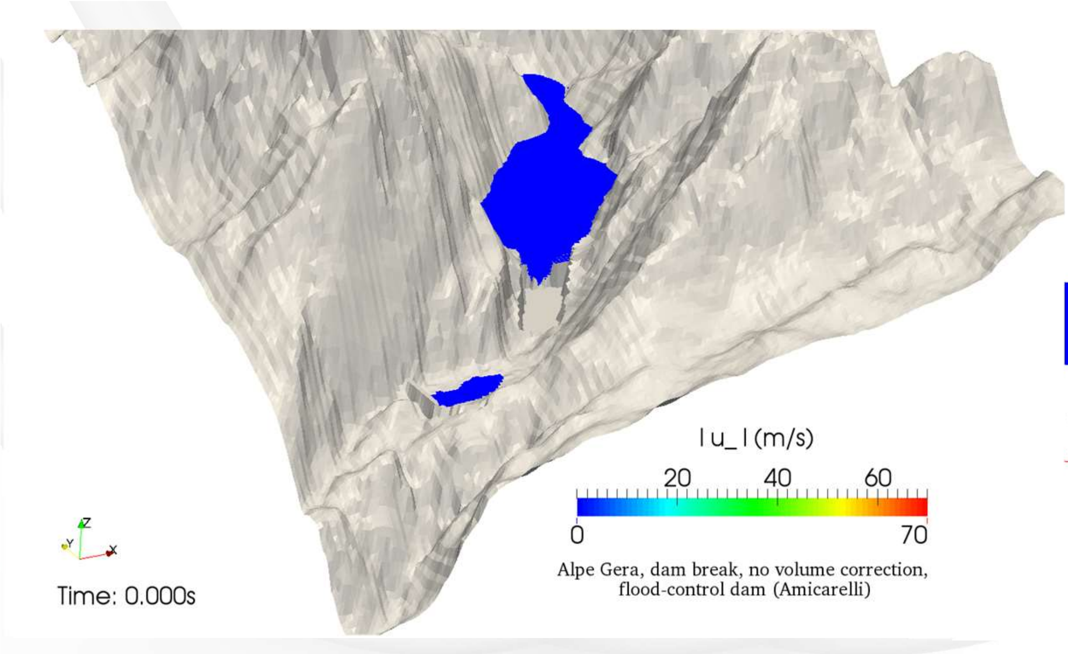
Amicarelli et al. (2017, IJCFD): scheme for dense granular flows

Amicarelli et al. (2015, CAF): scheme for body transport in free surface flows

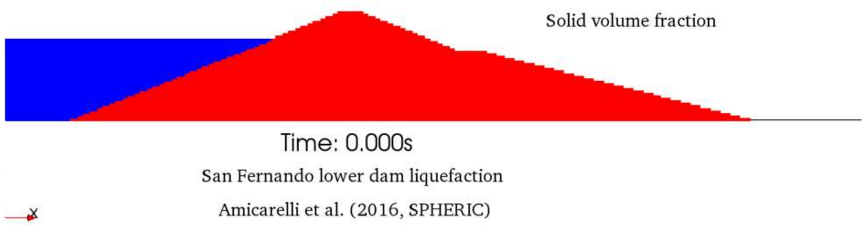
Amicarelli et al. (2013, IJNME): boundary treatment scheme (surface elements+LPRS)

Manenti et al. (2012, JHE): 2D erosion criterion

Di Monaco et al. (2011, EACFM): boundary treatment scheme (volume integrals)



Alpe Gera dam breaks (SPHERIC 2019)



San Fernando earth-dam liquefaction (SPHERIC 2016)

References and acknowledgments

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  - ☑ HSPHER9b (Amicarelli et al.)

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Thanks for your attention,
any correction/question is welcome
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