

Soil Mechanics for Unsaturated Soils

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Dedicated
to our parents
George and Esther Fredlund
and
Sugiarto and Pauline Rahardjo
who taught us that the fear of the Lord was the beginning of
wisdom and that the love of the Lord makes life worth living

Delwyn G. Fredlund
Harianto Rahardjo

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