

Groundwater and Seepage

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McGRAW-HILL BOOK COMPANY

New York
San Francisco
Toronto
London

GROUNDWATER AND SEEPAGE

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1617181920 VBVB 09876

ISBN 07-026740-5

these general areas is assumed on the part of the reader of this volume. Students have found the study of the mathematical concepts to be particularly rewarding: however, in keeping with an utilitarian philosophy, a conscientious attempt has been made to reduce solutions to simple graphs or charts.

A number of completely worked examples are provided, and over 200 problems of varying degrees of difficulty are included. These range from proofs of a routine nature to practical applications of the text material.

In writing a text of this type there arises the inevitable problem of selection. The author's first impulse was toward the general theory of groundwater and seepage, including both steady and transient states. However, with the growth of the present volume, it was decided to consider only steady-state flow and to defer the transient problem to another volume.

The author wishes to express his gratitude to his friend and colleague Dr. Gerald A. Leonards for his invaluable suggestions and discussions, and to Mrs. J. Becknell for typing the manuscript.

M. E. Harr

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